

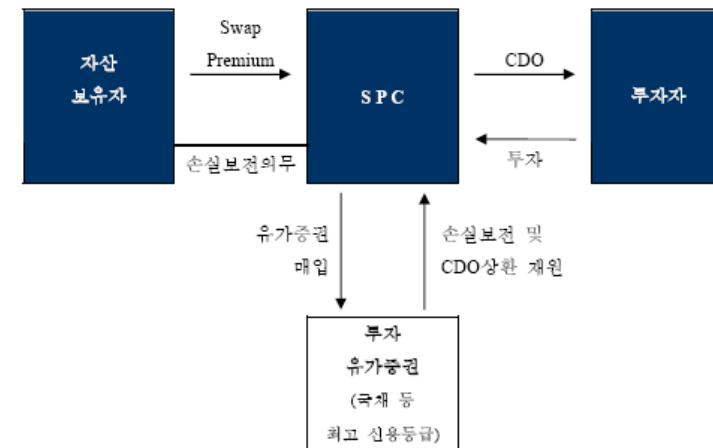
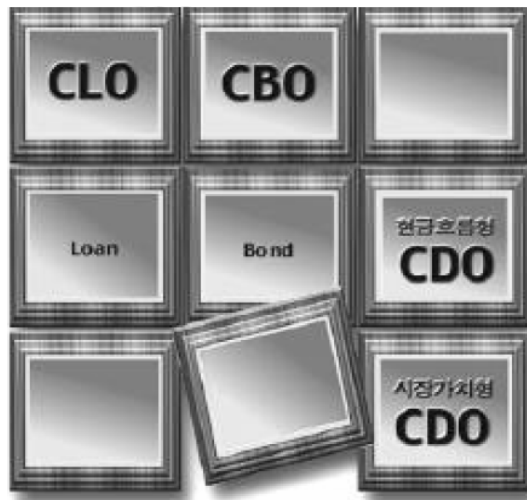


CDO 분석 및 리스크 측정 - Credit Risk & Valuation of CDO

이석형 (감사지원국 신용리스크반 선임 감사역)



- 1. What is a CDO
- 2. Cash Flow CDO structure
 - Waterfall
 - Coverage Test
- 3. Valuation & Credit Risk



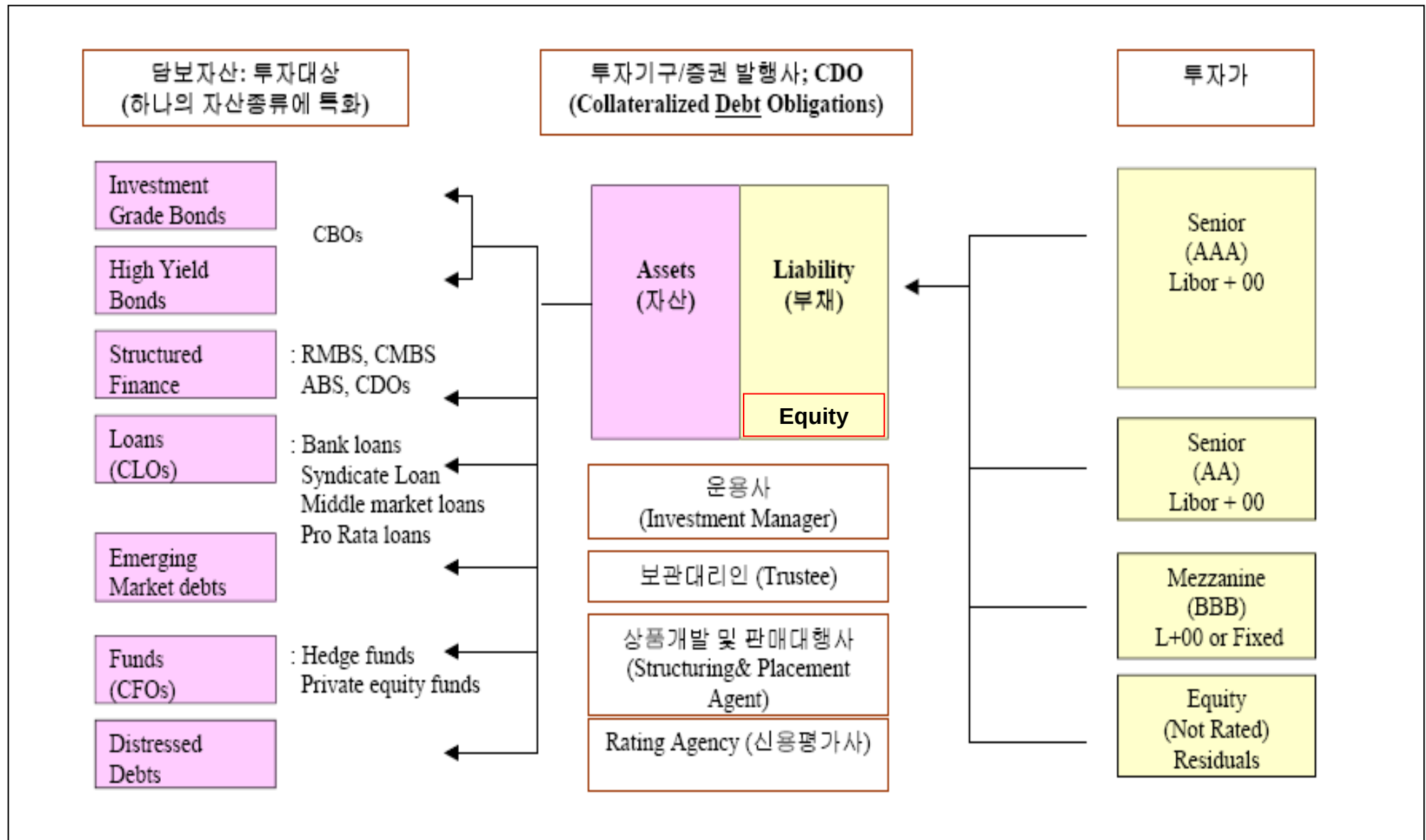
1. What is a CDO?

자산 담보부 증권(Collateralized Debt Obligation)

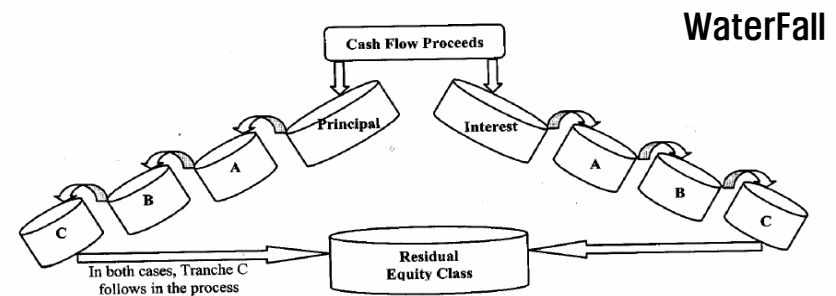
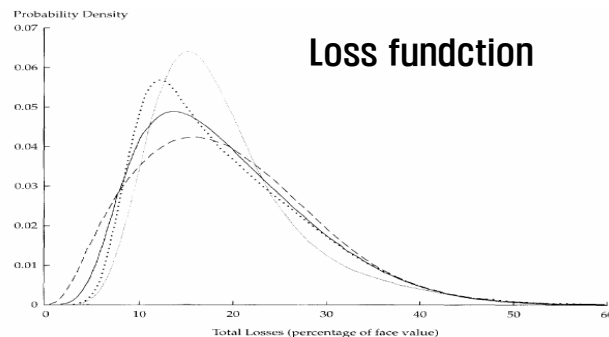
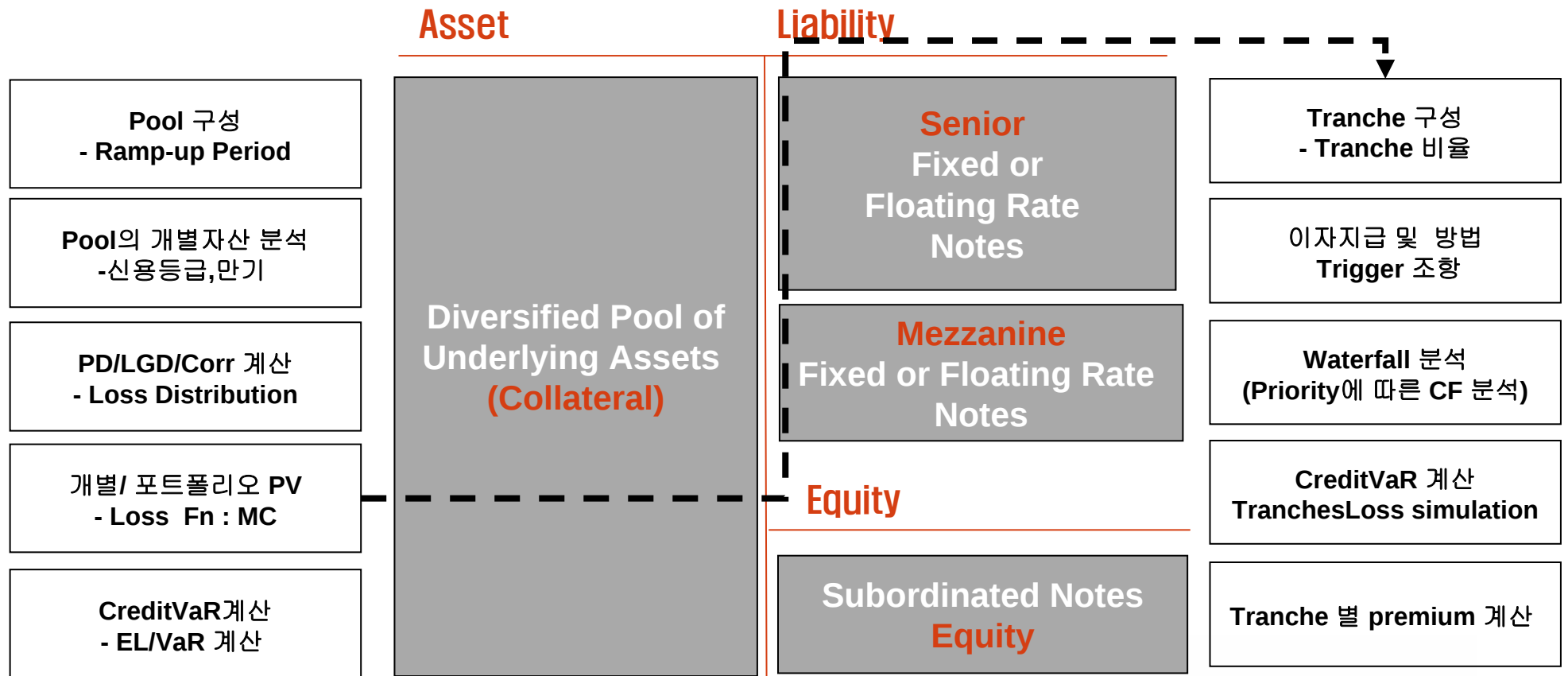
- 기초자산이 대출(**loan**)이나 채권(**Bond**)으로 이루어져 있는 증권

분류 기준	CDO 종류	특 징
기초자산의 종류	CBO CLO CDO Square	기초자산이 채권으로 이루어짐 기초자산이 대출채권으로만 이루어짐 다른 CDO들을 기초자산으로 함
기초자산 수익원천	CashFlow CDO Market Value CDO Synthetic CDO	
유동화 목적	Arbitrage CDO Balance sheet CDO	차익거래 위험전가
기타 분류	위험노출 Cash Synthetic Hybrid 운용정도 Active Static Slightly Managed 자산종류 CLO ABS CDO CBO Synthetic CDO TRUPS CDO CFO	

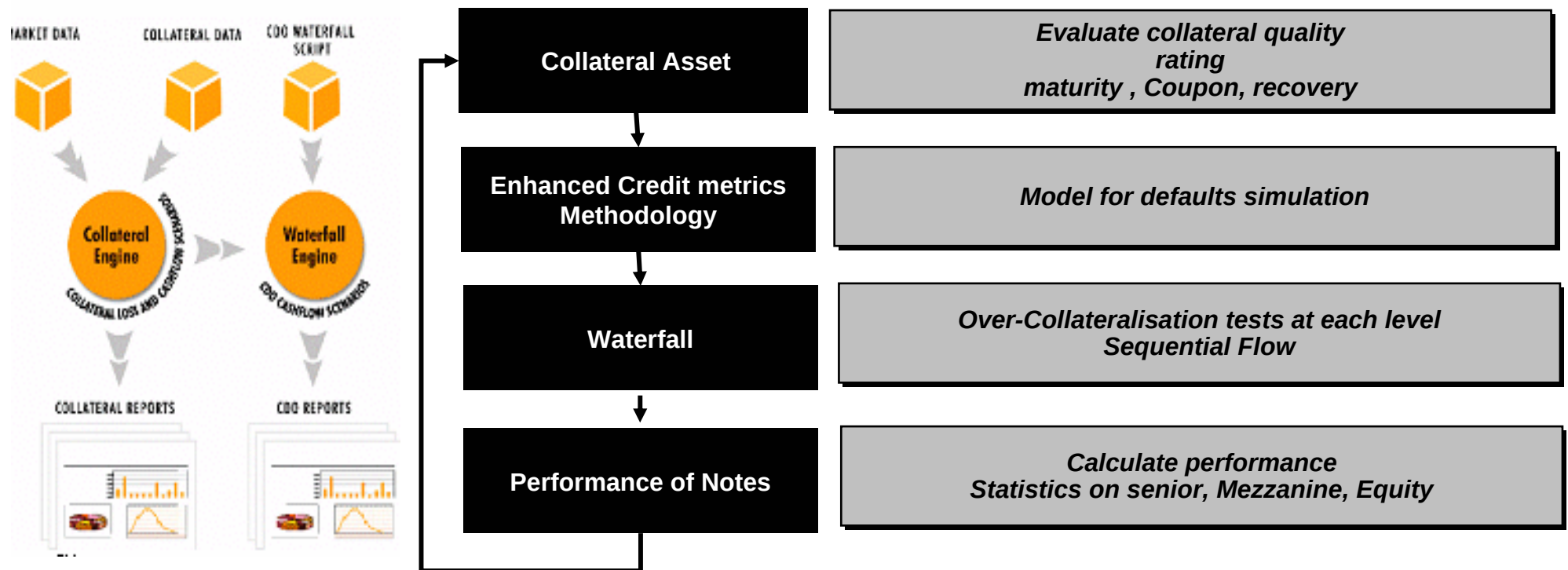
1. What is a CDO? : 일반적인 구조



1. What is a CDO? - 특징

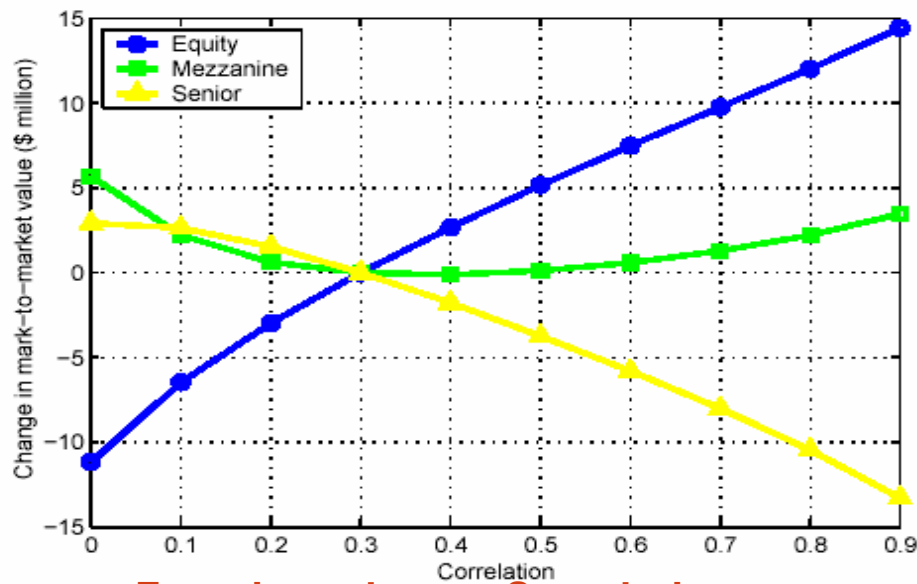


1. What is a CDO? : Analysis of CDO : CDO Flow

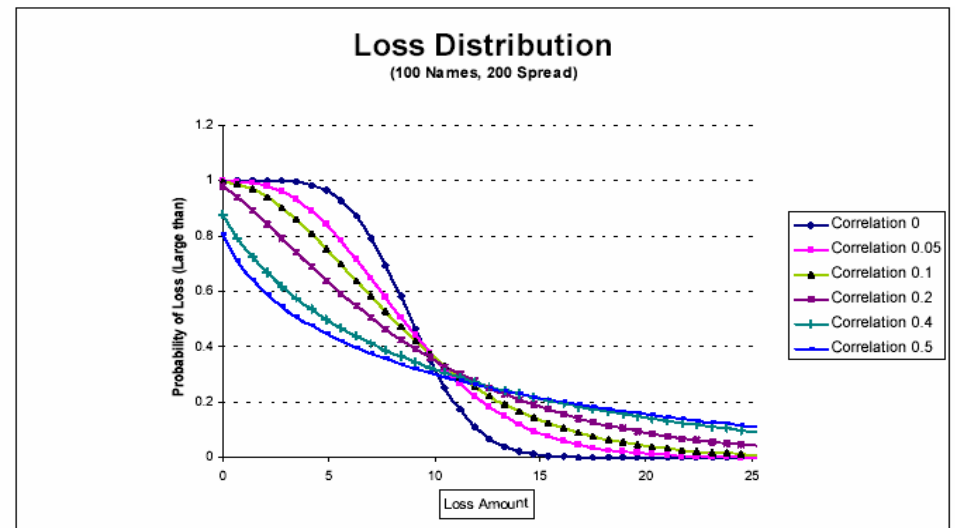


- Build a scenario of 'Default time' for each asset, thereby effecting timing of cash flows received, and delays in recovery.
- Generate scenarios of default times for each asset in the collateral pool using a Monte Carlo simulation process. Front loaded defaults analysis would be accounted for in some of the scenarios. Also, can focus on adverse scenarios.

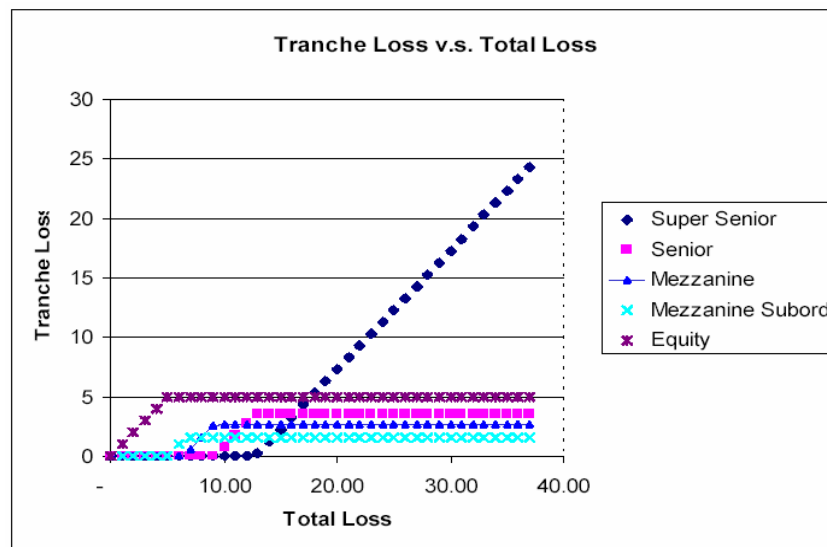
1. What is a CDO? : The effect of Main factor



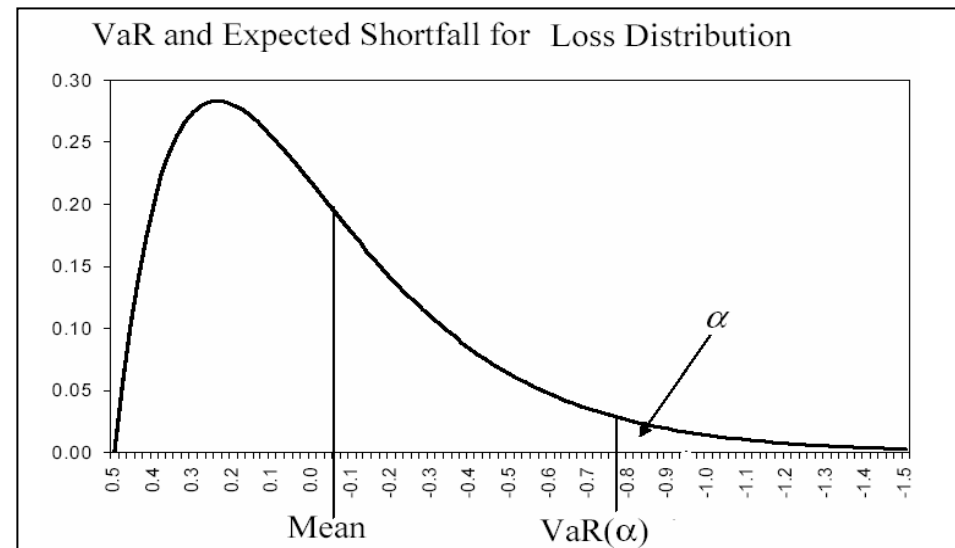
Tranche value as Correlation



Loss Distribution as Correlation



Total Loss of each Tranche



Loss distribution

1. What is a CDO? : Modeling

– EX) Model Inputs and Outputs

Capital Structure

Maturity (years) 5.14908676

Currency USD

Frequency 4

Note	Notional	Stated Spread (bps)	Fair Spread (bps)	Mark to Market
15% - 30% notes	150,000,000	15	3	856,439
10% - 15% notes	50,000,000	59	34	614,266
7% - 10% Notes	30,000,000	120	105	216,095
3% - 7% Notes	40,000,000	345	341	70,835
0% - 3% tranche	30,000,000	1,200	1,567	-3,701,066

The Mark-to-Market value is the discounted expectation of the payments that the protection seller receives minus the discounted expectation of the losses that the protection seller covers. The number is the actual value of the tranche in the currency specified from the point of view of the protection seller. Discounting is done at the risk-free rate.

The Fair Spread indicates the spread that makes the expected payments to the protection seller equal to the expected losses paid by the protection seller (i.e., the spread that makes the Mark-to-Market equal to zero). The number is rounded to the nearest basis point.

Collateral Pool

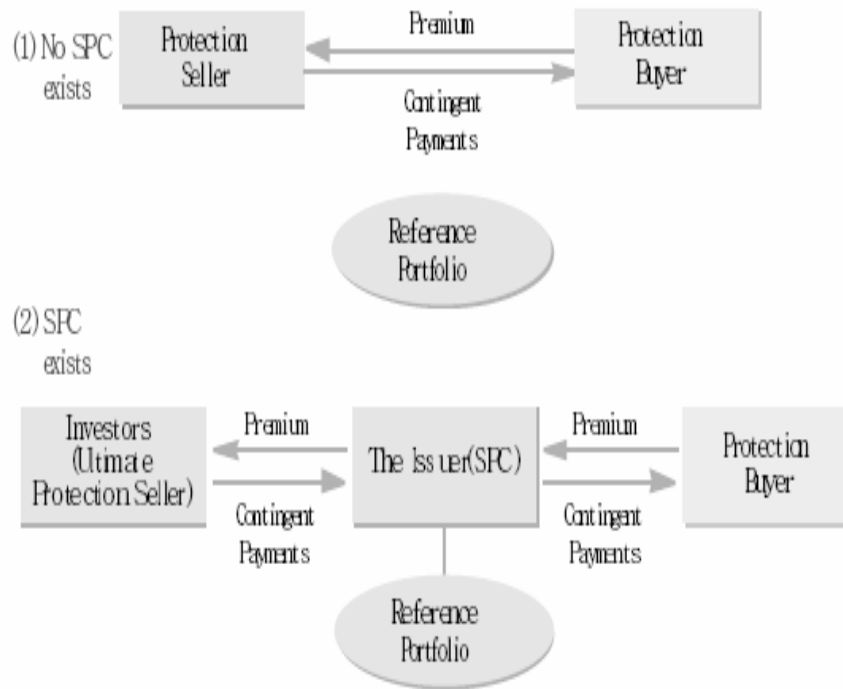
Correlation (%) 25 ☐ Use average correlation

Name	Notional	Recovery Rate	Spreads			MTM Change (+10bp Shift)				
			CDS	CreditGrade	Credit Rating	15% - 30% notes	10% - 15% notes	7% - 10% Notes	3% - 7% Notes	0% - 3% tranche
AOL Time Warner Inc.	10,000,000 USD	50.00%	48	181	50	-1,417	-3,970	-6,112	-18,517	-23,783
AT&T Corp.	10,000,000 USD	50.00%	95	268	65	-881	-2,850	-4,873	-17,005	-26,601
AT&T Wireless Services, Inc.	10,000,000 USD	50.00%	95	240	65	-881	-2,850	-4,873	-17,005	-26,601
Aetna Inc.	10,000,000 USD	50.00%	39	151	94	-1,599	-4,304	-6,438	-18,783	-22,929
Albertson's, Inc.	10,000,000 USD	50.00%	65	75	50	-1,171	-3,484	-5,604	-17,992	-25,024
Alcoa Inc.	10,000,000 USD	50.00%	22	33	28	-2,124	-5,169	-7,201	-19,178	-20,736
Altria Group, Inc.	10,000,000 USD	50.00%	165	23	65	-530	-1,957	-3,695	-14,820	-28,527
American Electric Power Company, Inc.	10,000,000 USD	50.00%	32	149	94	-1,792	-4,637	-6,746	-18,982	-22,078

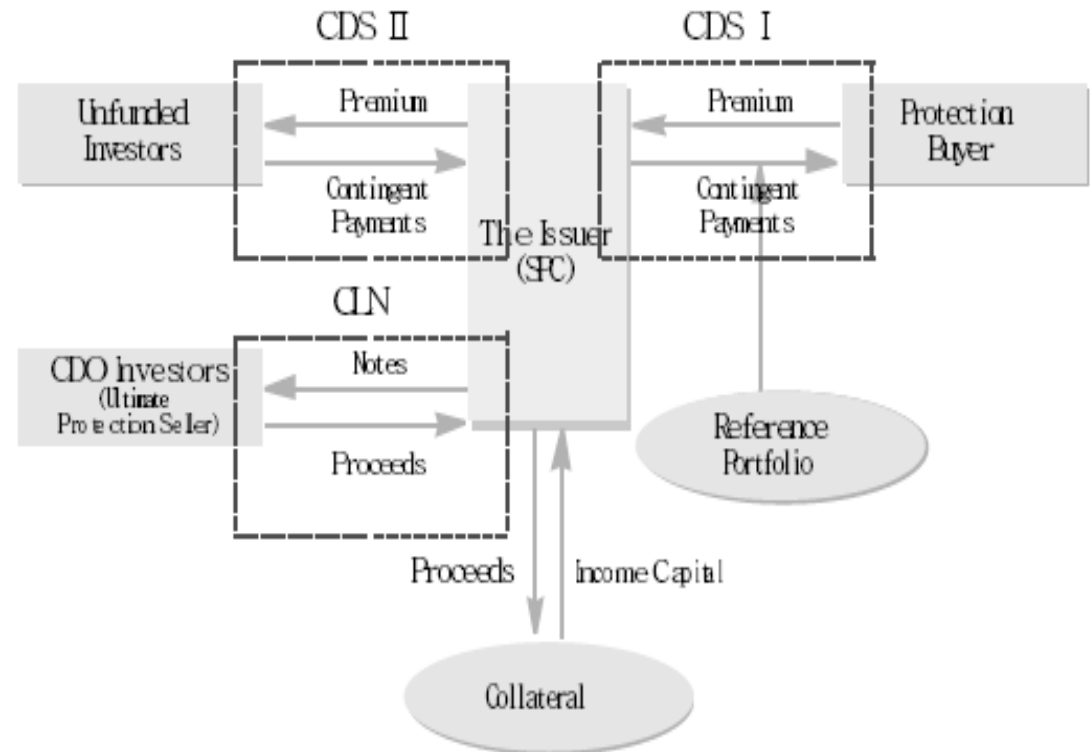
1. What is a CDO? : Synthetic CDO

- **Synthetic** : 신용파생상품을 활용하여 기초 자산군을 실제 양도하지 않고 투자자에게 신용위험을 이전
- Synthetic CDO는 **Balance Sheet/Arbitrage, Static/Managed** 등 여러 가지 기준에 따라 분류가 가능하다. 그 중 하나는 증권(Notes) 발행 혹은 CDS 체결여부에 따라 구분되는 **Unfunded/Partially Funded/Funded Synthetic CDO**이다.

Unfunded SCDO



Funded SCDO



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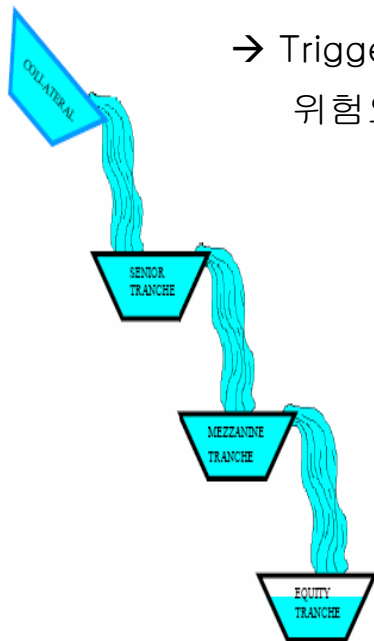
2.Cash Flow CDO structure -Waterfall

- 담보 Pool에서 발행되는 현금흐름으로 이자(interest)와 원금(principal)이 우선순위(priority)에 따라 지급.

Trigger : 기술적 신용보강(CE)수단

- Performance Trigger : 기초자산의 발생 및 회수 등의 성과저하
- Rating Trigger : 기관의 신용등급 하락
- Credit Event Trigger : 현금창출 능력을 크게 악화시키게 되는 예기치 못하는 신용사건이 발생

→ Trigger 장치를 마련하여 발행된 사채의 등급 하향 및 원리금 지급능력 하락 위험으로부터 투자자를 사전에 보호



유동화 거래에 있어 통상적으로 사용되는 ‘조기상환사유’라는 표현은 여러 가지 Trigger 조항들 가운데 대표적인 예

→ Trigger 상황에서 발효될 수 있는 다양한 위험 통제 방법들 가운데 유동화 증권의 조기상환(Early Amortization)이라는 조치를 예정하고 설정된 장치로서, 유동화 증권에 대한 신용위험으로 인하여 발행된 사채의 등급 저하 위험을 강력하게 저지할 수 있는 방어책

2.Cash Flow CDO structure - Coverage Tests

- 담보자산의 coverage tests는 각 tranche의 지급되는 현금흐름이 담보자산의 현금흐름으로 충분한지를 알아볼 수 있음.
- **Haircut test:** haircut collateral mark-to-market $\geq$ debt tranche par + accrued interest (haircut rate depends on asset class)
- **Over-collateralization (OC) test:** ratio of the total par collateral value to the sum of par value of the tranche and all tranches senior to this tranche $\geq$ threshold
- **Interest coverage (IC) test:** ratio of total collateral interest to the sum of interest on the tranche and tranches senior to this tranche $\geq$ threshold

예제)

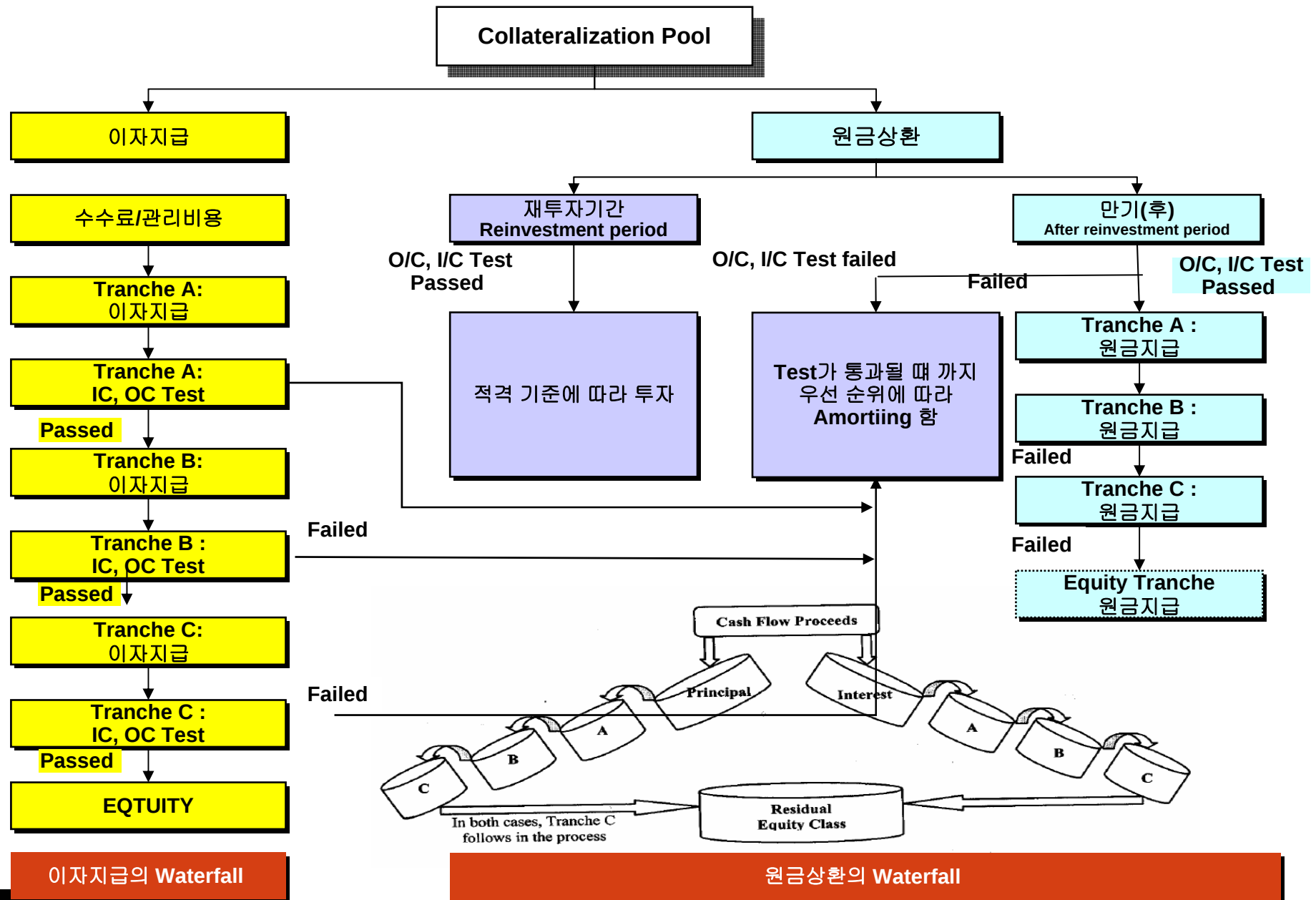
N = Collateral face value,

A = Face value of class A notes, B = Face value of class B notes

Tests: $N/A > q1(\text{tranche1})$
 $N/(A+B) > q2(\text{tranche2})$

If tests not satisfied at any coupon period, A and B are reduced by redeeming notes. Similar tests for interest payments.

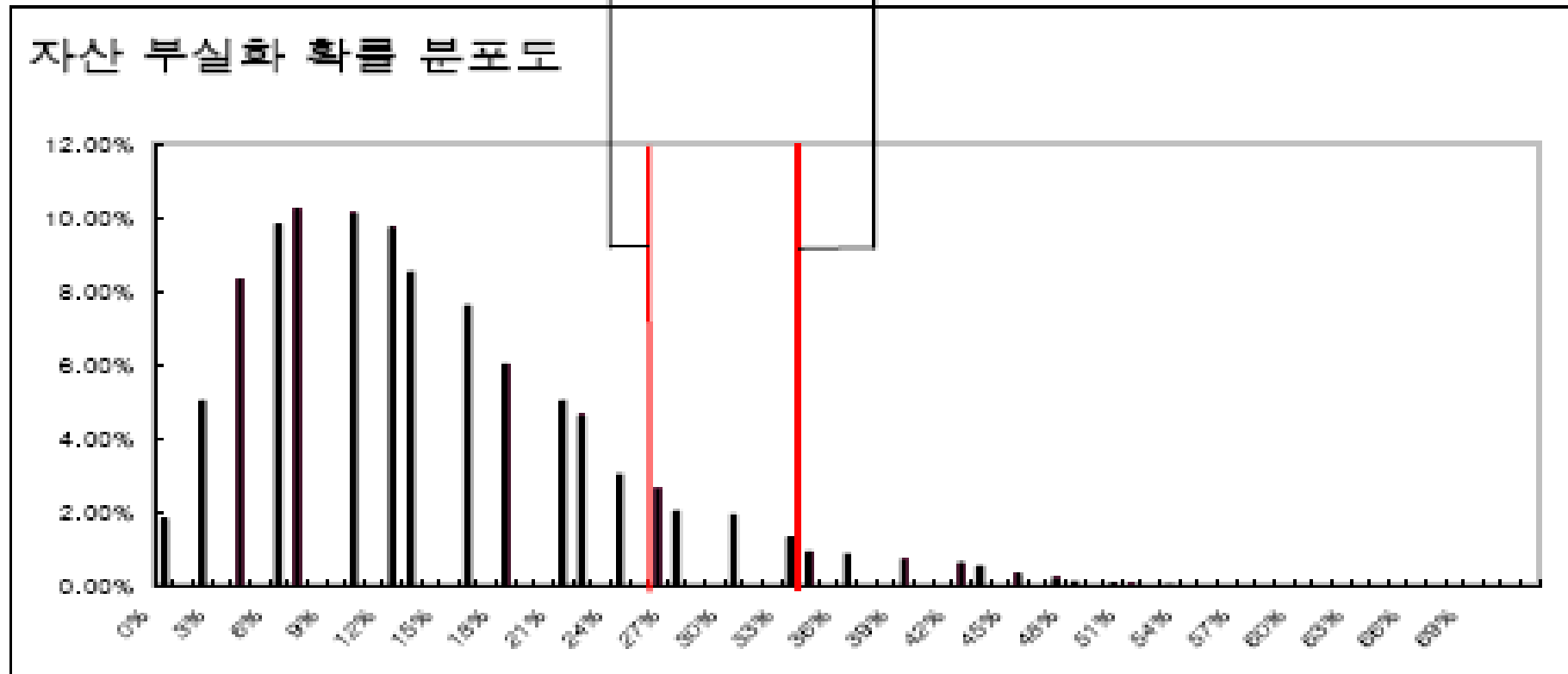
2. Cash Flow CDO structure -Waterfall



2. Cash Flow CDO structure -Waterfall

92.5th percentile = 28% 부실화

97th percentile = 36% 부실화



- X축 : 기초자산 Pool의 전체규모 중 부실화된 자산의 비중을 백분율로 표시
- Y축 : X축 상의 각 결과가 발생할 확률

2.Cash Flow CDO structure - Coverage Tests –O/C Test

Class	원금	Tranche 비율	신용등급	기준금리 대 Spread(bps)	최소 요구 O/C 비율	최소 요구 I/C 비율
A	225,000,000	75%	Aa2	100	120%	140%
B	30,000,000	10%	Baa3	250	110%	125%
C	15,000,000	5%	Ba1	550	105%	110%
Equity	30,000,000	10%	Not Rated			

O/C Test Class A Notes:

PAR_{pool} : 담보자산 포트폴리오의 원금상환액

$$\frac{300,000,000}{225,000,000} = \frac{300}{225} = 133\%$$

$$O / C_A = \frac{PAR_{Pool}}{PAR_A} \geq (O / C)_A^{\min} = 120\%$$

O/C Test Class B Notes:

$$\frac{300,000,000}{225,000,000 + 30,000,000} = \frac{300}{255} = 118\%$$

$$O / C_B = \frac{PAR_{Pool}}{PAR_A + PAR_B} \geq (O / C)_B^{\min} = 110\%$$

O/C Test Class C Notes:

$$\frac{300,000,000}{225,000,000 + 30,000,000 + 15,000,000} = \frac{300}{270} = 111\%$$

$$O / C_C = \frac{PAR_{Pool}}{PAR_A + PAR_B + PAR_C} \geq (O / C)_C^{\min} = 105\%$$

만약 이 비율이 발행 정관의 120% 가 넘으면 발행시 조건의 현금흐름과 같지만 그렇지 않을 경우는 우선순위에 따라 상환이 이루어 진다.

2.Cash Flow CDO structure - Coverage Tests – I/O Test

Class	원금	Tranche 비율	신용등급	기준금리 대 Spread(bps)	최소 요구 O/C 비율	최소 요구 I/C 비율
A	225,000,000	75%	Aa2	100	120%	140%
B	30,000,000	10%	Baa3	250	110%	125%
C	15,000,000	5%	Ba1	550	105%	110%
Equity	30,000,000	10%	Not Rated			

I/C Test Class A Notes:

$$\frac{(300,000,000 \times 10.4\% - 450,000) \times 0.5}{225,000,000 \times (4\% + 1\%) \times 0.5} = 273\%$$

Total Fees	450,000	USD p.a.
LIBOR	4.00%	Annahme!
WAC	10.40%	Pool Data!

WAC = Weighted Average Coupon

I/C Test Class B Notes:

$$\frac{(300,000,000 \times 10.4\% - 450,000) \times 0.5}{[225,000,000 \times (4\% + 1\%) + 30,000,000 \times (4\% + 2.5\%)] \times 0.5} = 233\%$$

I/C Test Class C Notes:

analog : 210%

$$I/C_A = \frac{PAR_{Pool} \times WAC - Fees}{PAR_A \times C_A}$$

$$I/C_B = \frac{PAR_{Pool} \times WAC - Fees}{PAR_A \times C_A + PAR_B \times C_B}$$

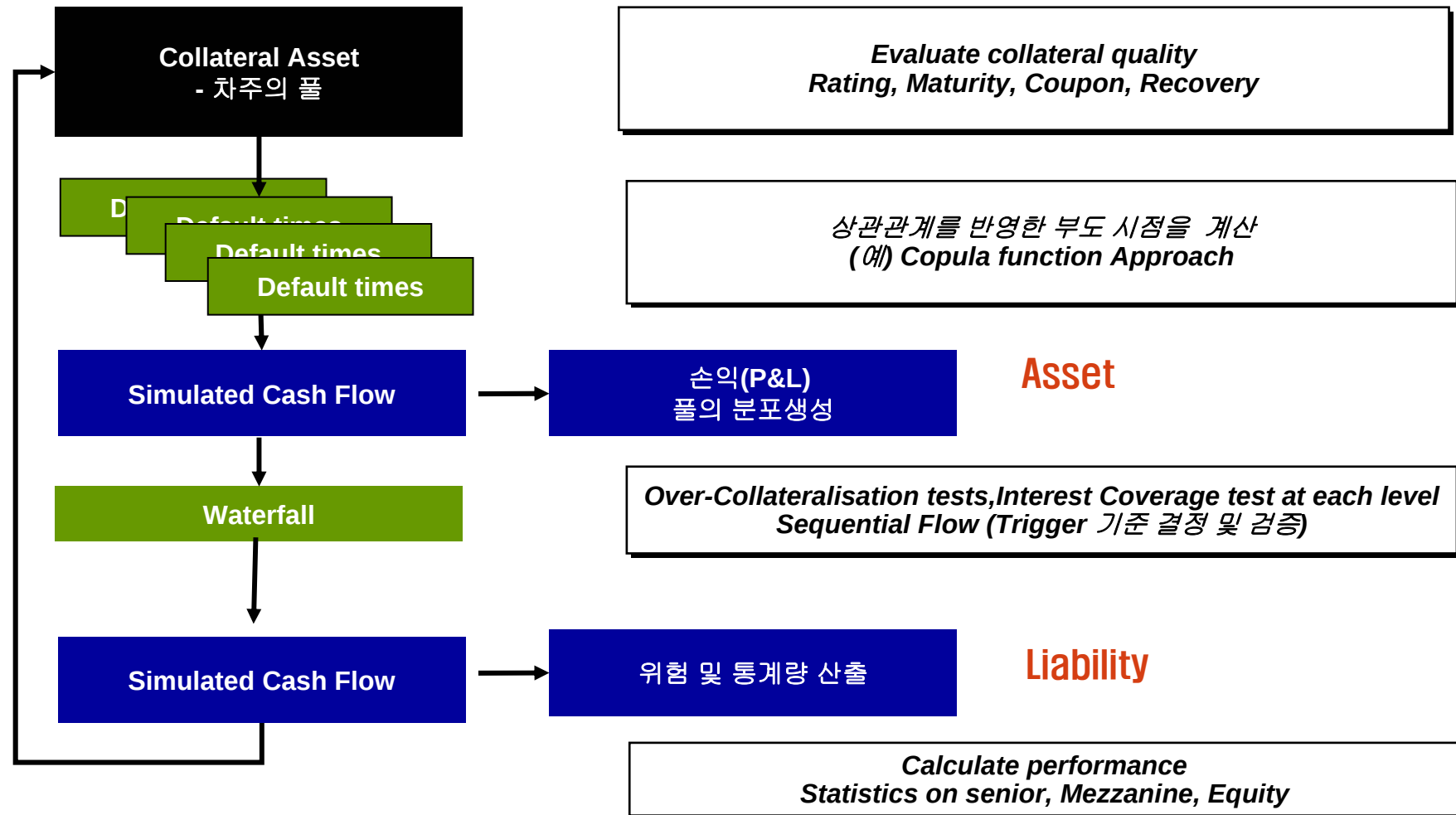
$$I/C_C = \frac{PAR_{Pool} \times WAC - Fees}{PAR_A \times C_A + PAR_B \times C_B + PAR_C \times C_C}$$

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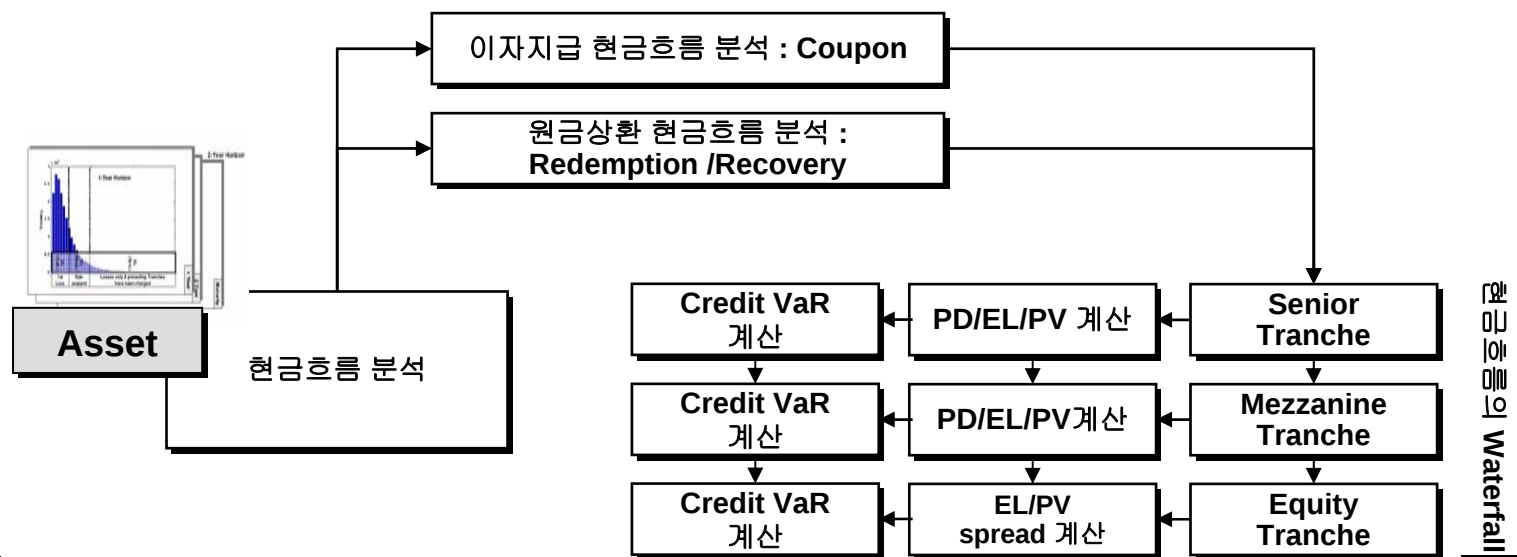
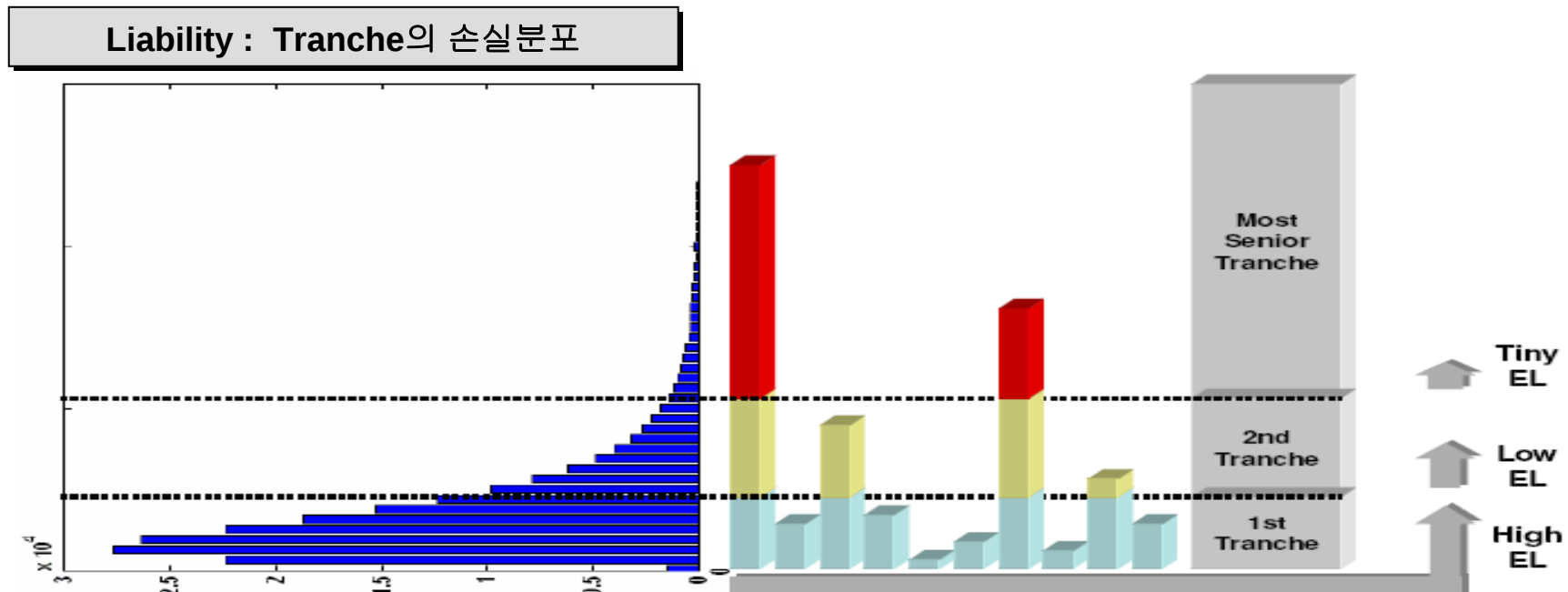


3. Valuation – Chart

- CDO의 가치를 결정하기 위하여 시뮬레이션 절차를 통하여 이루어짐.



1. What is a CDO? : Risk



3. Valuation - 부도상관관계

□ 부도상관관계 : 부도확률과 부도확률 사이의 상관관계

1. 역사적 Data이용

□ Hazard rate function : **Survival Time** 모형

2. Firm Value 모형

- 기업가치가 일정수준 이하로 하락한 경우 부도가 발행함
- Merton 모형 (Option Pricing 모형)

Survival Time 모형

$$q_A = P[D_A] \quad q_B = P[D_B] \quad q_{AB} = P[D_A D_B]$$

◆ 1년 동안 두 채권 A,B의 부도사건 : $D_A \quad D_B$

$$\rho = \frac{q_{AB} - q_A q_B}{\sqrt{q_A(1-q_A)q_B(1-q_B)}}$$

★ 임의적으로 선택된 특정기간의 부도상관관계를 나타내며 관측시점에 따라 달라지는 부도확률을 고려하지 못한다는 문제가 있음

→ **Survival Time** 모형

3. Valuation – 생존함수, Hazard Rate

□ 누적 부도 확률, 생존함수

$$F(t) = P(T \leq t) \quad t \geq 0$$

T : 부도 발생시점까지의 시간

$$S(t) = 1 - F(t) = P(T > t) \quad t \geq 0$$

S(t) : 생존함수 (Survival Function)

부도확률계산 : 신용평가회사의 부도율 이용, Merton 모델을 이용

□ Hazard rate : $h(t)$

$$P[t < T < t + \Delta t | T > t] = \frac{F(t + \Delta t) - F(t)}{1 - F(t)} \approx \frac{f(t)\Delta t}{1 - F(t)} = h(t)$$

- t 시점까지 생존했다고 할 때 지금 당장 부도가 발생할 확률

□ Default Process : hazard rate function을 찾아냄으로써 모형화 할 수 있게 되고 1년보다 짧은 기간에도 적용 가능

$$f(t) = S(t)h(t) \quad \frac{f(t)\Delta t}{1 - F(t)} = \frac{f(t)\Delta t}{S(t)} = h(t) \quad \Delta t = 1 \quad 1\text{년 기준}$$

□ Default Correlation : 생존기간간의 상관관계

$$\rho = \frac{\text{Cov}(T_A, T_B)}{\sqrt{\text{Var}(T_A)\text{Var}(T_B)}} = \frac{E(T_A T_B) - E(T_A)E(T_B)}{\sqrt{\text{Var}(T_A)\text{Var}(T_B)}}$$

3. Valuation – Survival rate, Hazard rate

□ 조건부 부도확률(Conditional Default Rate)

$${}_t q_x = \Pr[T - x \leq t | T > x]$$

${}_t q_x$: 조건부 부도확률 $\rightarrow$ x 기간 생존한 상태에서 기간 동안 부도날 확률

$${}_t p_x = 1 - {}_t q_x = \Pr[T - x > t | T > x]$$

예) ${}_t p_o = S(t)$ $\leftarrow$ 생존확률

$${}_t q_0 = \Pr[T - 0 \leq t | T > 0] = \Pr[T \leq t] = 1 - S(t) = F(t) \quad \leftarrow \text{누적 부도 확률}$$

□ Default Process

$$f(t) = S(t)h(t) \quad \frac{f(t)\Delta t}{1 - F(t)} = \frac{f(t)\Delta t}{S(t)} = -\frac{S(t)'}{S(t)} = h(t) \quad \Delta t = 1 \quad \text{1년 기준}$$

$$S(t) = e^{-\int_0^t h(s)ds}$$

$$F(t) = 1 - S(t) = 1 - e^{-\int_0^t h(s)ds}$$

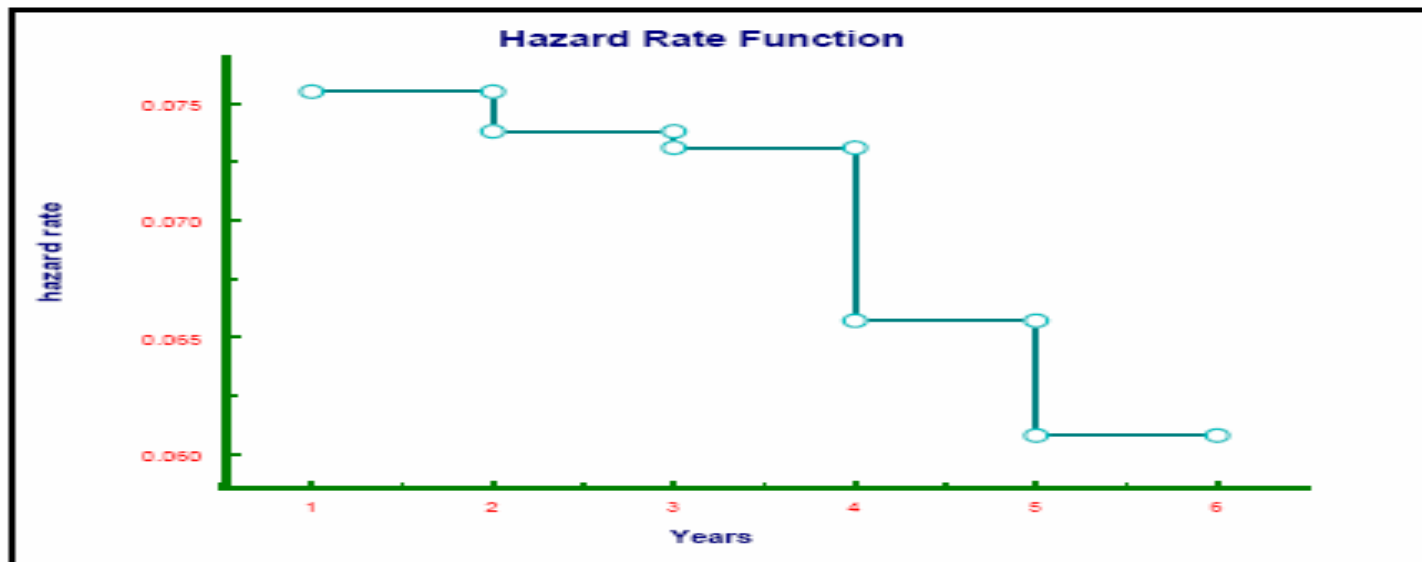
$${}_t P_x = e^{-\int_0^t h(s)ds}$$

$${}_t q_x = 1 - e^{-\int_0^t h(s)ds}$$

3. Valuation – Hazard rate 예시

Hazard rate의 계산 예시

Year	1	2	3	4	5
	1q0	2q0	3q0	4q0	5q0
누적부도확률	7.27%	13.87%	19.94%	25.03%	29.45%
S(t)	92.73%	92.88%	92.95%	93.64%	94.10%
h	7.55%	7.38%	7.31%	6.57%	6.08%



3. Valuation - Default Time

□ Default Time

$$N_\tau = I_{\{\tau \leq t\}} \quad \begin{aligned} I_{\{\tau \leq t\}} &= 1 & \text{if } \tau \leq t \\ I_{\{\tau \leq t\}} &= 0 & \text{if } \tau > t \end{aligned} \quad \tau : \text{부도시점}$$

□ Distribution of Default Time

$\{N_\tau(t)\}_{T>t \geq 0}$: 부도 수이며 이는 **hazard rate**를 모수로 하는 **Poisson Process**를 따름

$$P[N_T - N_t = k] = \frac{1}{k!} [h(T-t)]^k e^{-h(T-t)}$$

$$S(t) = e^{-\int_0^t h(s) ds}$$

$$F(t) = 1 - S(t) = 1 - e^{-\int_0^t h(s) ds}$$

$$\tau = \frac{-\ln U}{h(t)}$$

U : [0,1] 사이의 임의의 수 (uniform random variable)

3. Valuation – Loss function

- 부도 시 손실분포를 시점마다 계산

- Default distribution

$$F_i(t) = P(\tau_i \leq t)$$

- Default event correlation

$$\rho_{ij}(t) = \text{corr}(I_{\{\tau_i \leq t\}}, I_{\{\tau_j \leq t\}})$$

• I : indicator function

- i 자산의 부도시 손실

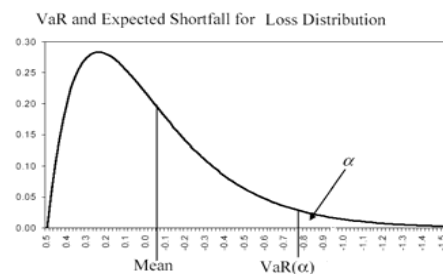
$$L_i(t) = (1 - R_i) N_i I_{\{\tau_i \leq t\}}$$

• R : 회수율, N : 발행금액

- Total 포트폴리오 손실

$$L(t) = \sum_{i=1}^n L_i(t)$$

• R : 회수율



3. Valuation – Loss function

•CDO의 경우에는 각 tranche의 손실은 각 tranche의 상위한계(upper bound)와 하위한계(lower bound)에 영향을 미침

$$K_{L_j} = a\% N_T < L(t) \leq b\% N_T = K_{U_j} \quad N_T = \sum_{i=1}^n N_i$$

□ Tranche의 손실

$$\begin{aligned} L_{A,B}(t) &= [L(t) - K_{U_j}] I_{\{L(t) \in [K_{U_j}, K_{L_j}]\}} + (K_{U_j} - K_{L_j}) I_{\{L(t) \in [K_{U_j}, N_T]\}} \\ &= \min[L(t), K_{U_j}] - \min[L(t), K_{L_j}] \end{aligned}$$

□ Tranche의 손실률

$$\frac{L_{a,b}(t)}{N} = [\ell(t) - a] I_{\{\ell(t) \in [a, b]\}} + (b - a) I_{\{\ell(t) \in [b, 100]\}}$$

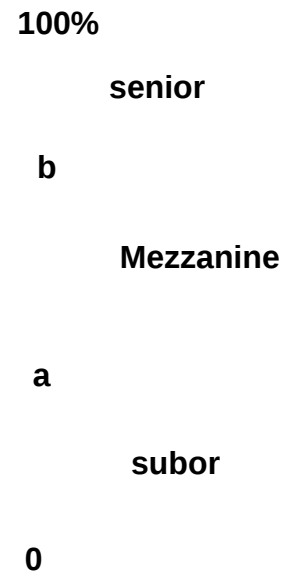
$$a < \ell(t) \leq b \quad \ell(t) = L(t) / N$$

3. Valuation – Loss function

□ 담보자산의 손실에 따른 각 **Tranche**의 손실량

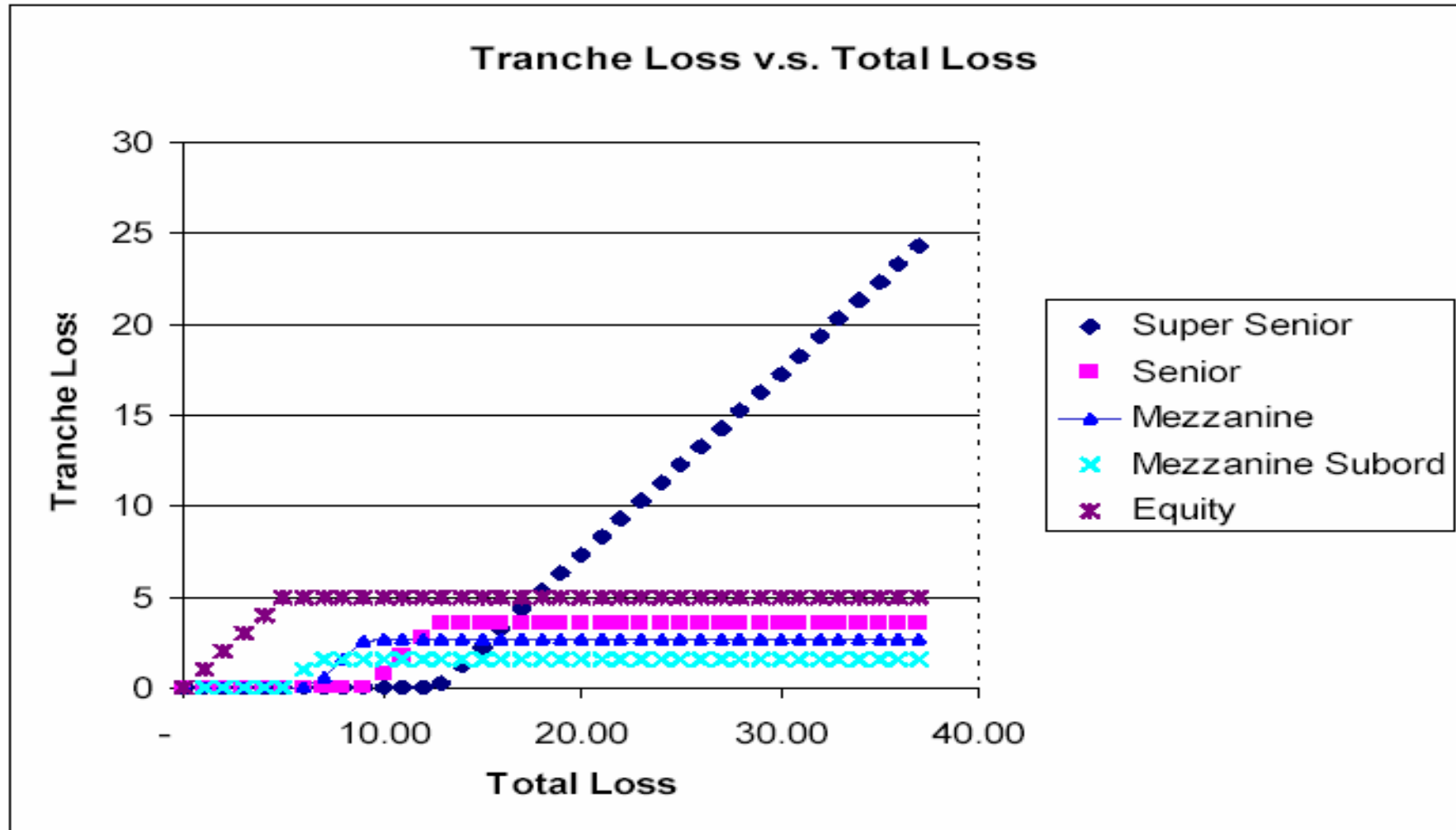
	Mezzanine	Senior	Junior or Equity
$\ell(t) < a$	0	0	$\ell(t)$
$a < \ell(t) < b$	$\ell(t) - a$	0	a
$\ell(t) > b$	$b - a$	$\ell(t) - b$	a

□



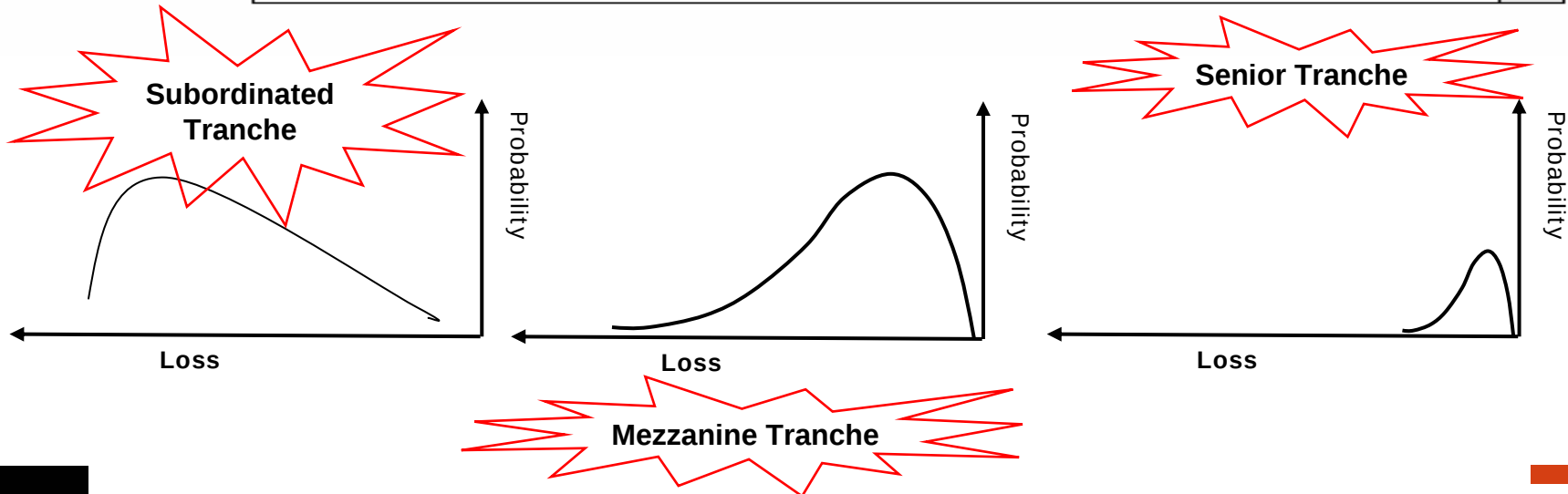
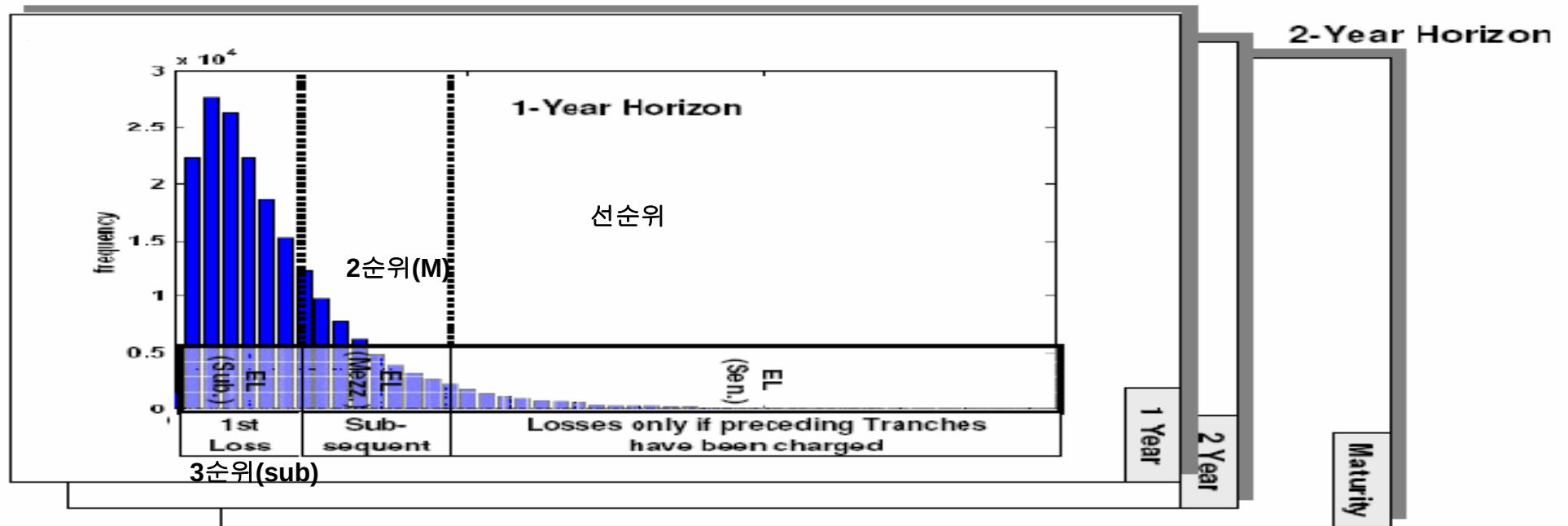
3. Valuation – Loss function

- Tranches are serviced according to a prioritization scheme:



3. Valuation – Risk

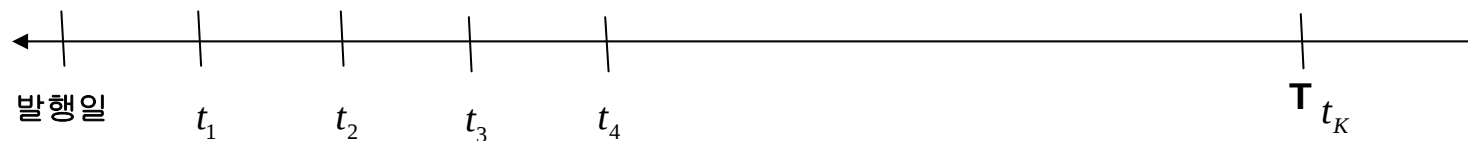
Asset : 풀의 손실 분포



3. Valuation - 각 Tranche의 EAD 계산

□ 현가 계산

$$\begin{aligned}
 \text{Pool의 가치}_{j,t} &= (K_{U_j} - K_{L_j})N_T - \ell(t)N_T = (K_{U_j} - K_{L_j} - \ell(t))N_T \\
 &= \begin{cases} (K_{U_j} - K_{L_j})N_T & \text{if } \ell(t) < K_{L_j} \\ (K_{U_j} - \ell(t))N_T & \text{if } K_{L_j} < \ell(t) \leq K_{U_j} \\ 0 & \text{if } \ell(t) > K_{U_j} \end{cases}
 \end{aligned}$$



S 인 프리미엄은 변하지 않으나 각 트랜치의 잔액은 손실로 인하여 시점마다 변한다.

Receive	$s_j \eta \text{Pool의 가치}_{j,t}$
기대손실	$(\ell_j(t) - \ell_j(t - \eta))N_T$

3. Valuation - CDO tranche의 가격

공정가격

• Premium Leg

$$\sum_{k=1}^K \frac{1}{(1+r)^{(t_k-t_0)}} s_j \eta E[(K_{U_i} - K_{L_i} - Z_{j,t_k}) N_T]$$

□ Z : 전체 포트폴리오의 손실을, n 이자 지급주기

• Default Leg

$$\sum_{k=1}^K \frac{1}{(1+r)^{(t_k-t_0)}} E[(Z_{j,t_k} - Z_{j,t_{k-1}}) N_T]$$

□ Marginal 손실률

Default leg(expected loss) = Premium Leg (premium 지급)

$$s_j = \frac{\sum_{k=1}^K \frac{1}{(1+r)^{(t_k-t_0)}} (E[Z_{j,t_k}] - E[Z_{j,t_{k-1}}])}{\sum_{k=1}^K \frac{1}{(1+r)^{(t_k-t_0)}} \eta E[(K_{U_i} - K_{L_i} - Z_{j,t_k})]}$$

3. Valuation - Simulation

Asset

N 차주의 Random number 생성

부도시간(Default Time) 계산

전체 담보 자산의 손실 분포 추정

담보 풀의 PV, EL, Credit VaR 계산

Liability

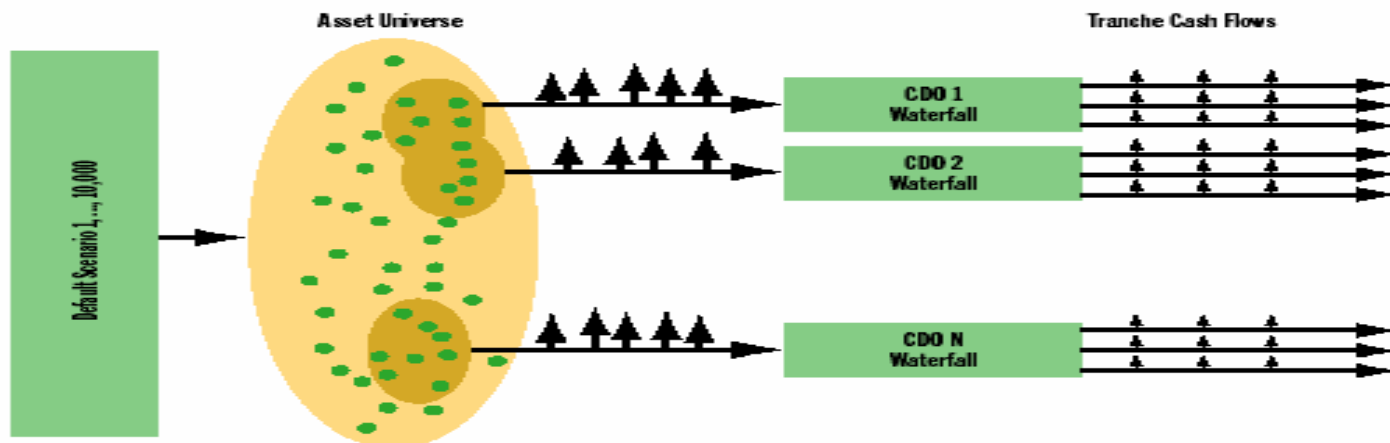
Waterfall 분석

Tranche별 손실함수 추정

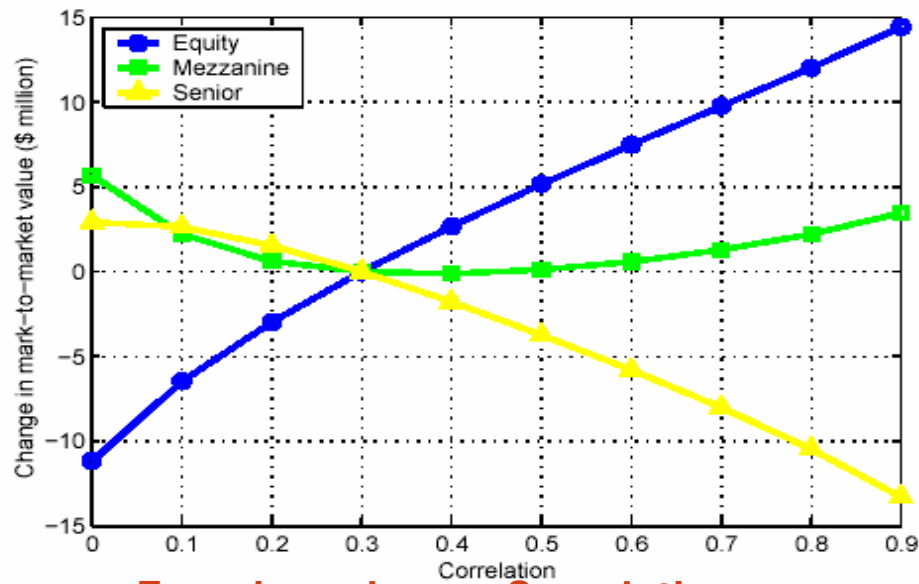
기대손실 = 프리미엄 계산

Credit VaR 계산

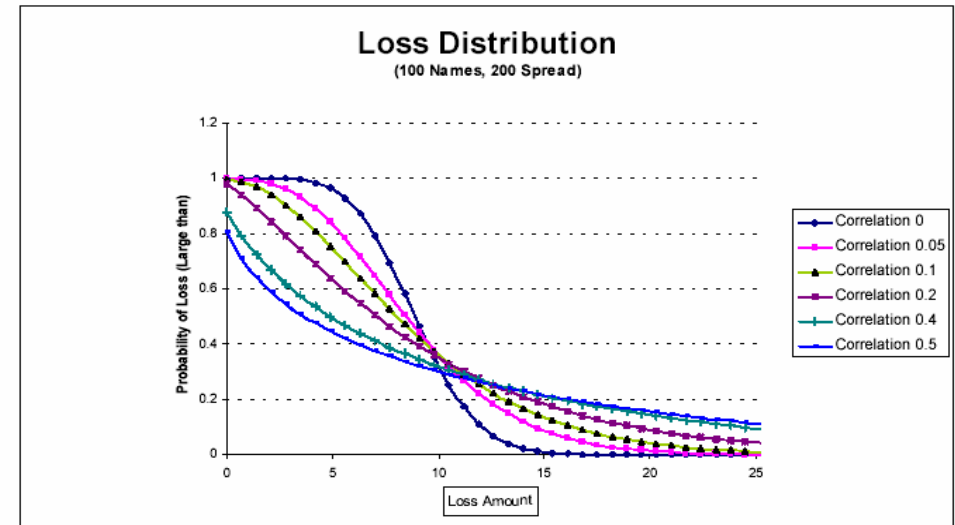
CDO Analysis



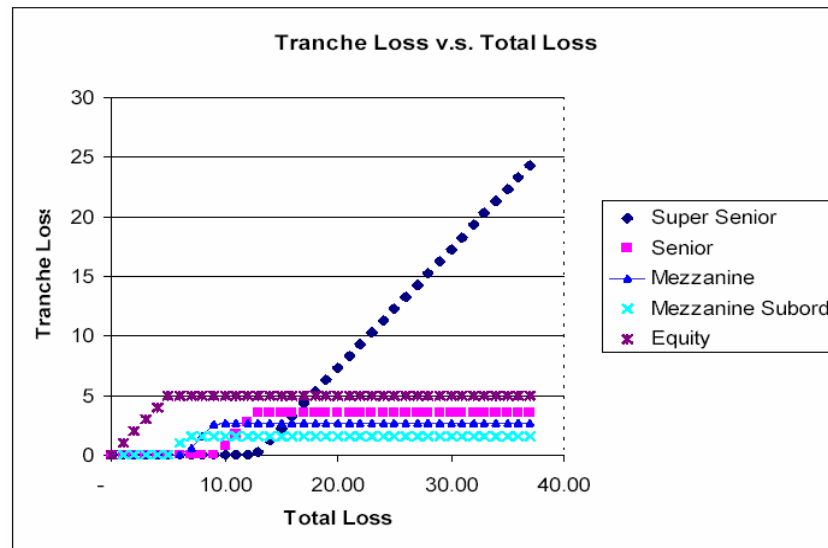
3. Valuation - Simulation



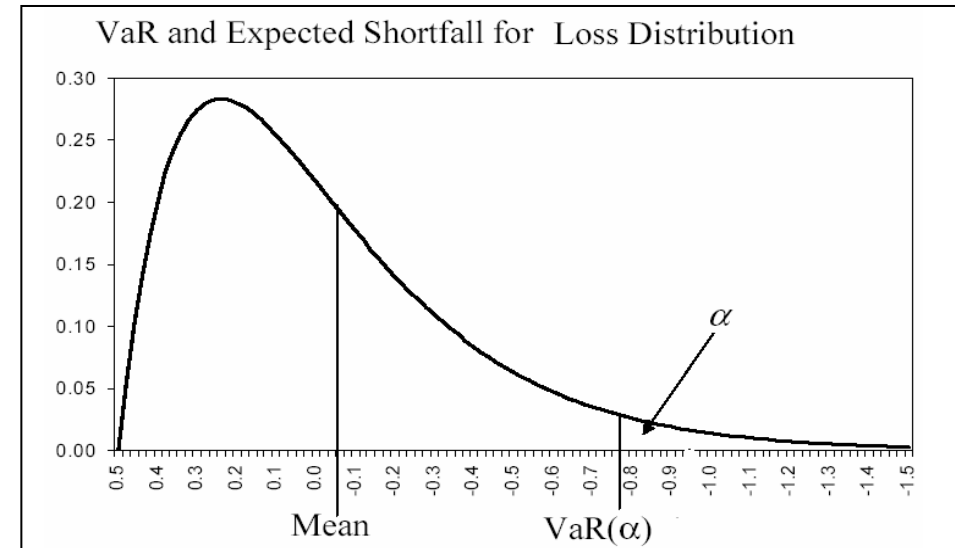
Tranche value as Correlation



Loss Distribution as Correlation



Total Loss of each Tranche



Loss distribution

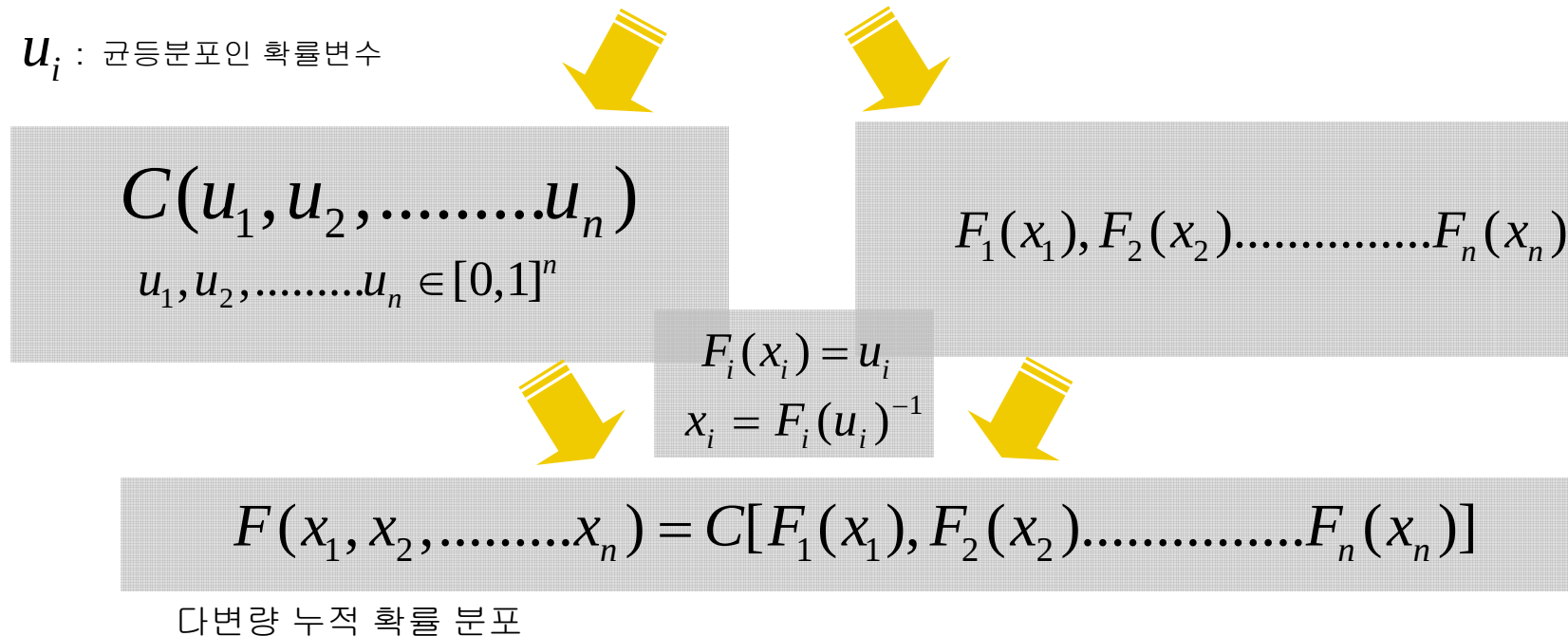
3. Valuation - simulation

포트폴리오의 함수 : Correlation 반영

Copula 함수 : 다변량 균등분포(Uniform Distribution) 확률 변수의 결합 누적 확률 분포

$$C(u_1, u_2, \dots, u_n) = \Pr[U_1 \leq u_1, U_2 \leq u_2, \dots, U_n \leq u_n]$$

u_i : 균등분포인 확률변수



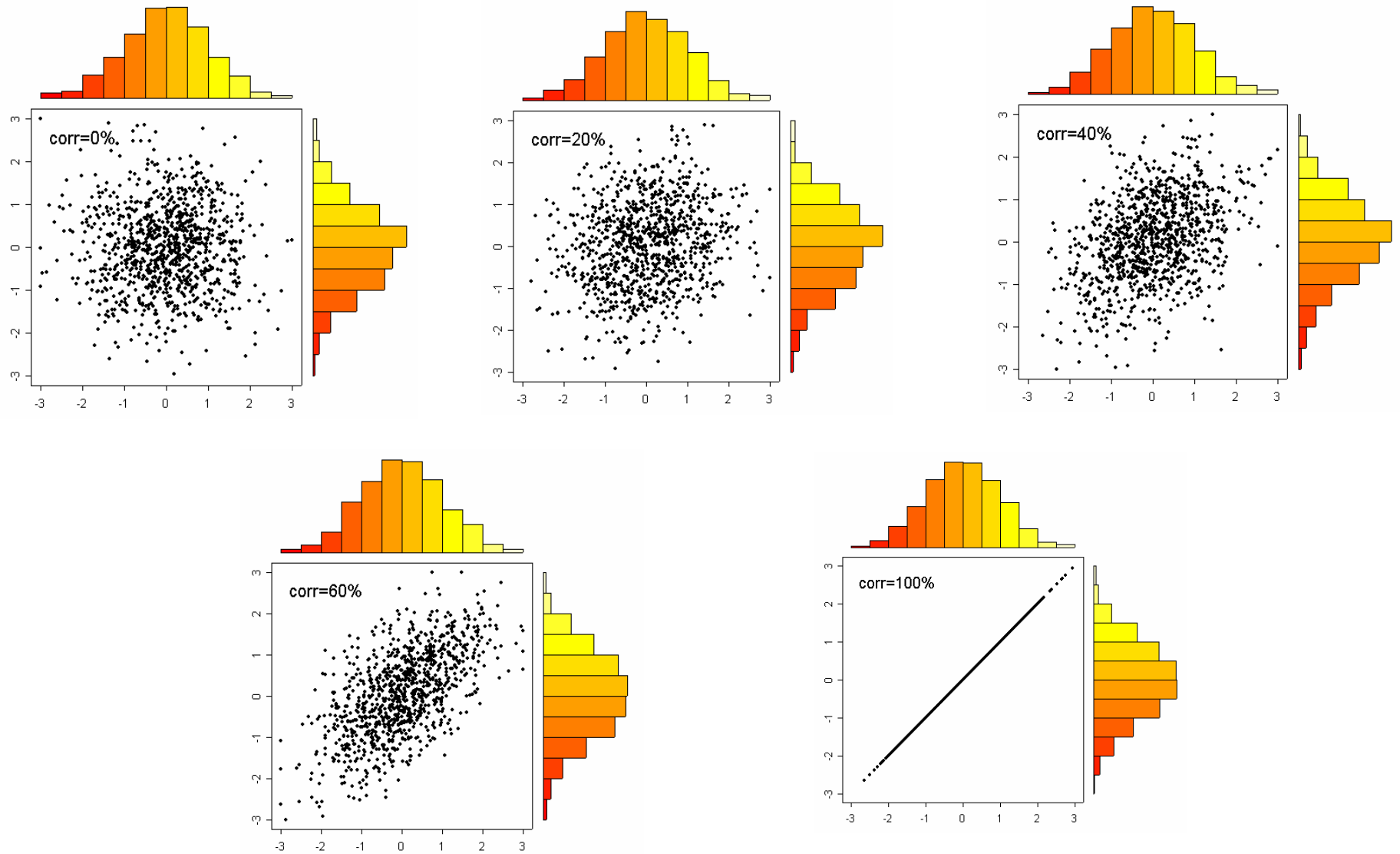
다변량 누적 확률 분포

Sklar Representation 정리

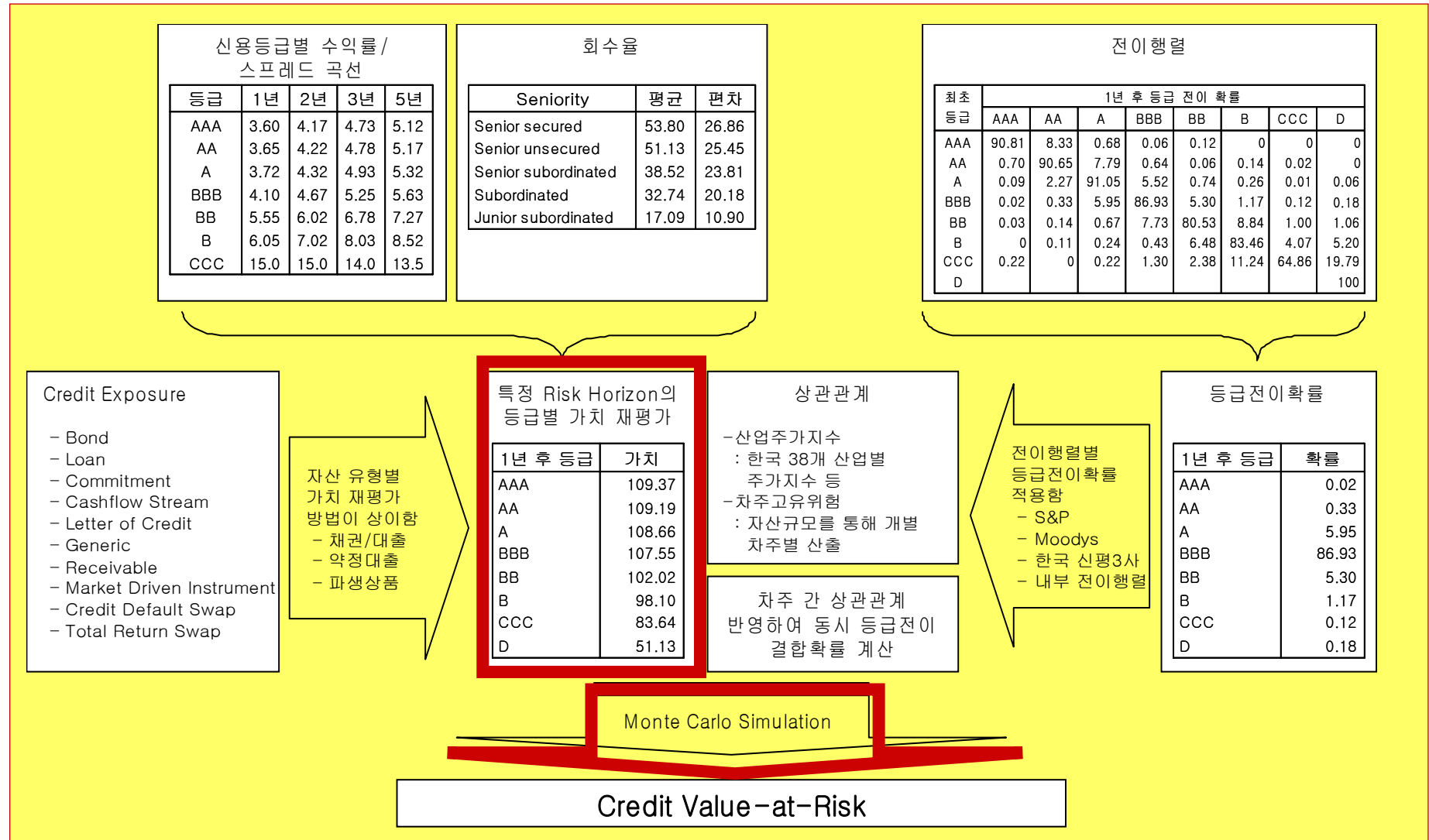
$$C(x_1, x_2, \dots, x_n) = F[F_1^{-1}(x_1), F_2^{-1}(x_2), \dots, F_n^{-1}(x_n)]$$

3. Valuation - simulation

상관관계에 따른 두 변수의 분포



3. Valuation - Calculating Credit VaR for CDO Tranches



Q & A