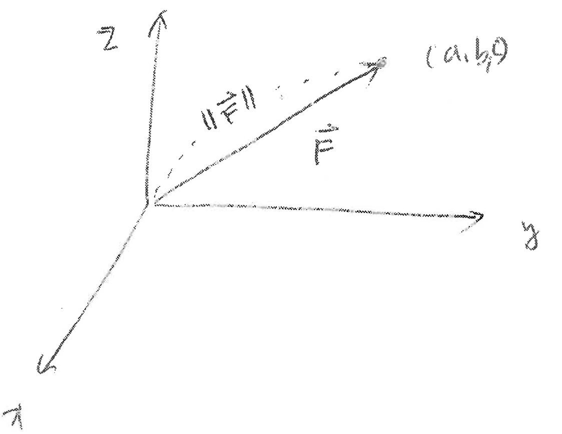
4.1

스칼라 : 크기

벡터 : 크기 & 방향.

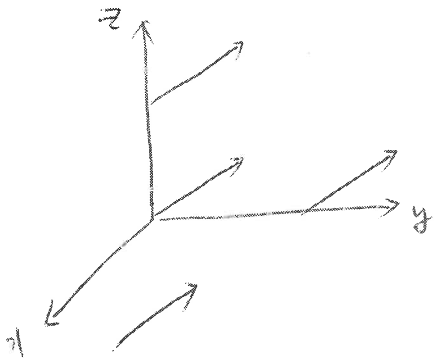


- 위치벡터 $(a,b,c) = \vec{r}$
- 원점으로부터 본 화살표 : 벡터의 방향.
- 길이 : 벡터의 크기 : norm (놈)

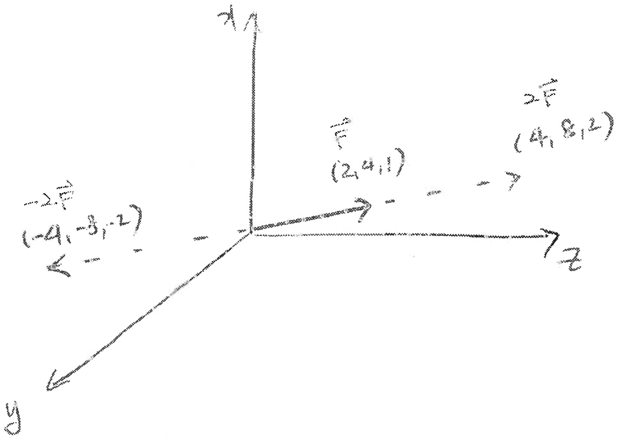
$$\|\vec{r}\| = \sqrt{a^2+b^2+c^2}$$

- 0 벡터 = $(0,0,0) = \mathbb{0}$

: 원점으로부터 화살표로 표시할 수 없는 유일한 벡터.



: 동일한 벡터 : 크기와 방향이 같은벡터.



: 평행 (parallel)

i) 벡터의 스칼라곱.

$\vec{F} = (a, b, c)$, 스칼라 α

$\alpha \vec{F} = (\alpha a, \alpha b, \alpha c)$

ii) 벡터의 합

$\vec{F} = (a_1, b_1, c_1)$, $\vec{G} = (a_2, b_2, c_2)$

$\vec{F} \pm \vec{G} = (a_1 \pm a_2, b_1 \pm b_2, c_1 \pm c_2)$

iii) 벡터의 대수

① $\vec{F} + \vec{G} = \vec{G} + \vec{F}$: 벡터합에 관한 교환법칙

② $(\vec{F} + \vec{G}) + \vec{H} = \vec{F} + (\vec{G} + \vec{H})$: 결합법칙

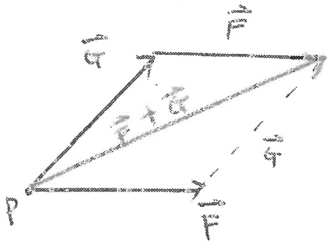
③ $\vec{F} + \vec{0} = \vec{F}$

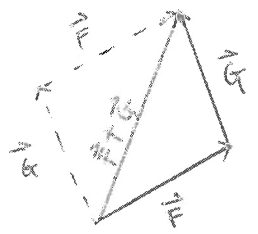
④ $\alpha(\vec{F} + \vec{G}) = \alpha\vec{F} + \alpha\vec{G}$

⑤ $(\alpha\beta)\vec{F} = \alpha(\beta\vec{F})$

⑥ $(\alpha + \beta)\vec{F} = \alpha\vec{F} + \beta\vec{F}$

\# 벡터합에 대한 평행사변형의 법칙 (Parallelogram law)





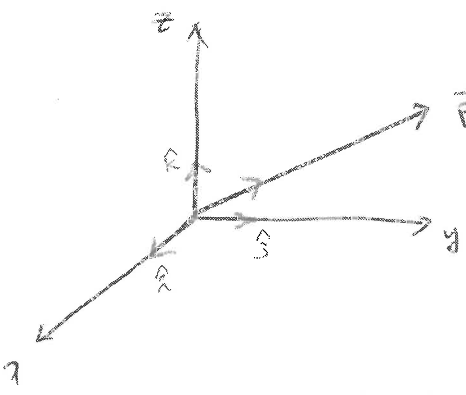
\# $\vec{F}$의 표준 표현식 (standard representation)

$$\vec{F} = (-3, 0, 1) = -3\hat{i} + \hat{k}$$

\# 단위벡터 (Unit Vectors) : 크기(norm)가 1인 벡터.

$\hat{i} = (1,0,0) = \hat{x}$, $\hat{j} = (0,1,0) = \hat{y}$, $\hat{k} = (0,0,1) = \hat{z}$

$\vec{F}$ 방향의 단위벡터 : $\dfrac{1}{\|\vec{F}\|}\vec{F}$



$\vec{F}(2,-3,1) = 2\hat{i} - 3\hat{j} + \hat{k}$

$$\|\vec{F}\| = \sqrt{(2)^2 + (-3)^2 + (1)^2} = \sqrt{14}$$

* $\vec{F}$ 방향의 단위벡터 : $\dfrac{\vec{F}}{\|\vec{F}\|} = \dfrac{1}{\sqrt{14}}(2, -3, 1)$

내적 (Dot Product)

$\vec{F} = (a_1, b_1, c_1)$ $\qquad$ $\vec{G} = (a_2, b_2, c_2)$

$\vec{F} \circ \vec{G} = a_1a_2 + b_1b_2 + c_1c_2$ $\quad$: 스칼라

내적의 성질

① $\vec{F} \circ \vec{G} = \vec{G} \circ \vec{F}$

② $(\vec{F} + \vec{G}) \circ \vec{H} = \vec{F} \circ \vec{H} + \vec{G} \circ \vec{H}$

③ $\alpha (\vec{F} \circ \vec{G}) = (\alpha \vec{F}) \circ \vec{G} = \vec{F} \circ (\alpha \vec{G})$

④ $\vec{F} \circ \vec{F} = \|\vec{F}\|^2$

⑤ $\vec{F} \circ \vec{F} = 0 \iff \vec{F} = 0$

⑥ $\|\alpha \vec{F} + \beta \vec{G}\|^2 = \alpha^2 \|\vec{F}\|^2 + 2\alpha\beta \vec{F} \circ \vec{G} + \beta^2 \|\vec{G}\|$

⑦ $\vec{F} \circ \vec{G} = \|\vec{F}\| \|\vec{G}\| \cos\theta$

$\iff \cos\theta = \dfrac{\vec{F} \circ \vec{G}}{\|\vec{F}\| \|\vec{G}\|}$

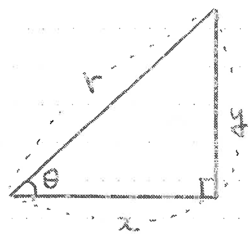
⑧ $\vec{F} \perp \vec{G} \iff \vec{F} \circ \vec{G} = 0$

⑨ Cauchy - Schwarz 부등식

$|\vec{F} \circ \vec{G}| \le \|\vec{F}\| \|\vec{G}\|$

$-1 \le \cos\theta \le 1$

〈삼각함수〉



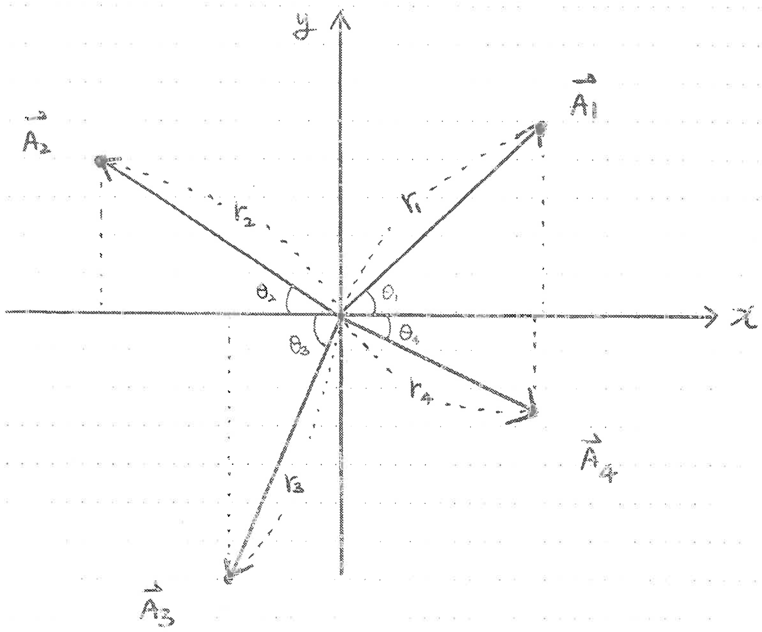
$$r^2 = x^2 + y^2$$

$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

$$\tan\theta = \frac{y}{x}$$

ex1)



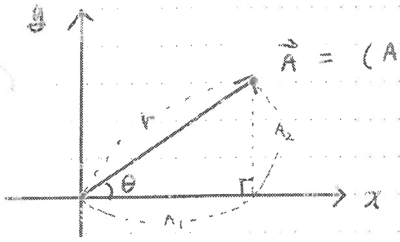
$$\vec{A_1} = (r_1\cos\theta_1 , r_1\sin\theta_1)$$

$$\vec{A_2} = (-r_2\cos\theta_2 , r_2\sin\theta_2)$$

$$\vec{A_3} = (-r_3\cos\theta_3 , -r_3\sin\theta_3)$$

$$\vec{A_4} = (r_4\cos\theta_4 , -r_4\sin\theta_4)$$

ex2)



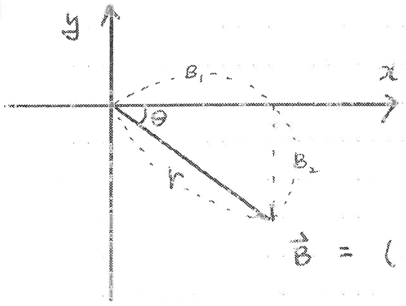
$\vec{A} = (A_1, A_2) = A_1 \hat{x} + A_2 \hat{y} = r\cos\theta\ \hat{x} + r\sin\theta\ \hat{y}$

$r^2 = A_1^2 + A_2^2 \Rightarrow r = \sqrt{A_1^2 + A_2^2}$

$\cos\theta = \dfrac{A_1}{r}$, $\sin\theta = \dfrac{A_2}{r}$, $\tan\theta = \dfrac{A_2}{A_1}$

$A_1 = r\cos\theta$, $A_2 = r\sin\theta$

ex3)



$\vec{B} = (B_1, -B_2) = B_1 \hat{x} - B_2 \hat{y} = r\cos\theta\ \hat{x} - r\sin\theta\ \hat{y}$

$r = \sqrt{B_1^2 + B_2^2}$

$\cos\theta = \dfrac{B_1}{r} \Rightarrow B_1 = r\cos\theta$

$\sin\theta = \dfrac{B_2}{r} \Rightarrow B_2 = r\sin\theta$

외적 (Cross Product)

$\vec{F} = (a_1, a_2, a_3) \qquad \vec{G} = (b_1, b_2, b_3)$

$$\vec{F} \times \vec{G} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \hat{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \hat{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \hat{k}$$: 벡터

$$= (a_2 b_3 - a_3 b_2)\hat{i} - (a_1 b_3 - a_3 b_1)\hat{j} + (a_1 b_2 - a_2 b_1)\hat{k}$$

외적의 성질

① $\vec{F} \times \vec{G} = -\vec{G} \times \vec{F}$

② $\vec{F} \times \vec{G} \perp \vec{F}$ 와 $\vec{G}$: 오른손 법칙 (right-hand rule) :

$\hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i} \qquad \hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j} \qquad \hat{i} \times \hat{k} = -\hat{j}$

③ $\|\vec{F} \times \vec{G}\| = \|\vec{F}\| \|\vec{G}\| \sin\theta$

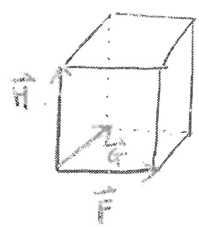
④ $\vec{F}$와 $\vec{G}$ 가 영벡터가 아니라고 하면,

$\vec{F} \times \vec{G} = 0 \iff \vec{F} \parallel \vec{G}$

⑤ $\vec{F} \times (\vec{G} + \vec{H}) = \vec{F} \times \vec{G} + \vec{F} \times \vec{H}$

⑥ $\alpha(\vec{F} \times \vec{G}) = (\alpha\vec{F}) \times \vec{G} = \vec{F} \times (\alpha\vec{G})$

⑦ $\|\vec{F} \times \vec{G}\|$: 평행사변형의 넓이



⑧ $|\vec{H} \cdot (\vec{F} \times \vec{G})|$: 평행육면체의 부피

※ $\vec{H} \cdot (\vec{F} \times \vec{G})$: 스칼라 삼중적 (Scalar triple produce)