Three-Tangle in Non-inertial Frame

Mi-Ra Hwang[1], Eylee Jung[2] and DaeKil Park[1,3]

[1] *Department of Electronic Engineering, Kyungnam University, ChangWon 631-701, Korea*

[2] *Center for Superfunctional Materials, Department of Chemistry, Pohang University of Science and Technology, San 31, Hyojadong, Namgu, Pohang 790-784, Korea*

[3] *Department of Physics, Kyungnam University, ChangWon 631-701, Korea*

Abstract

Let Alice, Bob, and Charlie initially share an arbitrary fermionic three-qubit pure state, whose three-tangle is $\tau_3^{(0)}$. It is shown that if one party among the three of them moves with a uniform acceleration with respect to the other parties, the three-tangle reduces to $\tau_3^{(0)} \cos^2 r$, where r denotes a statistical factor in Fermi-Dirac statistics.

Recently, quantum information theories in the relativistic framework have attracted considerable attention[1–3]. It seems to be mainly due to the fact that many modern experiments on quantum information processing involve the use of photons and/or electrons, where the relativistic effect is not negligible. Furthermore, relativistic quantum information is also important from purely theoretical aspects[4] in the context of black hole physics and quantum gravity.

It has been shown in Ref.[3] that the entanglement formed initially in an inertial frame is generally degraded in a non-inertial frame. In particular, the bipartite bosonic entanglement completely vanishes when one of the two parties approaches the Rindler horizon. However, for the tripartite case there is a remnant of bosonic entanglement even in the Rindler horizon.

The fermionic entanglement can be addressed more easily than its bosonic counterpart due to the Pauli exclusion principle. By making use of the Unruh effect[5] in the fermionic qubit system and computing the Bogoliubov coefficients explicitly, one can show that within the single mode approximation, the vacuum state $|0\rangle_M$ and the one-particle state $|1\rangle_M$, where the subscript M stands for Minkowski space, in a uniformly accelerated frame with acceleration a are transformed into

$$\begin{aligned} |0\rangle_M &\rightarrow \cos r|0\rangle_I|0\rangle_{II} + \sin r|1\rangle_I|1\rangle_{II} \\ |1\rangle_M &\rightarrow |1\rangle_I|0\rangle_{II}, \end{aligned} \tag{1}$$

where the parameter r is defined by

$$\cos r = \frac{1}{\sqrt{1+\exp(-2\pi\omega c/a)}}, \tag{2}$$

and c denotes the speed of light, and ω represents the central frequency of the fermion wave packet. It is to be noted that the sign in the denominator of Eq. (2) is originated from Fermi-Dirac statistics. In Eq. (1) $|n\rangle_I$ and $|n\rangle_{II}$ $(n = 0, 1)$ indicate the mode decomposition in two causally disconnected regions in Rindler space.

In this paper we will examine the three-tangle[6], one of the most important tripartite entanglement measures, in a non-inertial frame. For this purpose we assume that Alice, Bob, and Charlie initially share an arbitrary pure three-qubit state $|\psi\rangle_{ABC}$. Applying a Schmidt decomposition we can transform $|\psi\rangle_{ABC}$ into the following[7];

$$|\psi\rangle_{ABC} = \lambda_0|000\rangle + \lambda_1 e^{i\varphi}|100\rangle + \lambda_2|101\rangle + \lambda_3|110\rangle + \lambda_4|111\rangle \tag{3}$$

where $\lambda_i \geq 0 \;\; (i = 0, \cdots, 4)$, $\sum_i \lambda_i^2 = 1$, and $0 \leq \varphi \leq \pi$. The three-tangle of $|\psi\rangle_{ABC}$ is $\tau_3^{(0)} = 4\lambda_0^2\lambda_4^2$. We will show in our study that if one party moves with a uniform acceleration a with respect to the other parties, the resulting three-tangle is degraded to

$$\tau_3 = \tau_3^{(0)} \cos^2 r, \tag{4}$$

regardless of which of the parties is accelerating. Therefore, at the Rindler horizon, the three-tangle reduces to half of the initial three-tangle.

Now, we assume that Alice, Bob, and Charlie were initially in the spacetime region I. If Alice was chosen as the accelerating party, the Unruh effect (1) transforms $|\psi\rangle_{ABC}$ into a four-qubit state $|\psi_A\rangle_{IBC\otimes II}$, whose explicit expression is

$$\begin{aligned} |\psi_A\rangle_{IBC\otimes II} = & \Big[\lambda_0 \cos r|000\rangle + \lambda_1 e^{i\varphi}|100\rangle + \lambda_2|101\rangle + \lambda_3|110\rangle + \lambda_4|111\rangle\Big] \otimes |0\rangle_{II} \\ & + \lambda_0 \sin r|100\rangle_{IBC} \otimes |1\rangle_{II}. \end{aligned} \tag{5}$$

Since all parties cannot access region II due to the causally disconnected condition, it is reasonable to take a partial trace over II to average the effect of qubit $|n\rangle_{II}$ out in Eq. (5). As a result, the initial state $|\psi\rangle_{ABC}$ reduces to a mixed state $\rho_{IBC} = \mathrm{tr}_{II}|\psi_A\rangle\langle\psi_A|$. This means that some information formed initially in region I is leaked into region II. Thus, Alice's acceleration induces an information loss, which is a main consequence of the Unruh effect.

One can show that the final state ρ_{IBC} is a rank-2 tensor in a form

$$\rho_{IBC} = p|a_+\rangle\langle a_+| + (1-p)|a_-\rangle\langle a_-| \tag{6}$$

where $p = (1 + \sqrt{\Delta})/2$ with

$$\Delta = \left(1 - 2\lambda_0^2 \sin^2 r\right)^2 + 4\lambda_0^2\lambda_1^2 \sin^2 r. \tag{7}$$

In Eq. (6) the vectors $|a_\pm\rangle$ are given by

$$|a_\pm\rangle = \frac{1}{\mathcal{N}_\pm}\Big[\lambda_0 \cos r z_\pm|000\rangle + y_\pm|100\rangle + \lambda_2 z_\pm|101\rangle + \lambda_3 z_\pm|110\rangle + \lambda_4 z_\pm|111\rangle\Big] \tag{8}$$

where

$$\begin{aligned} z_\pm &= 1 - 2\lambda_0^2 \sin^2 r \pm \sqrt{\Delta} \\ y_\pm &= e^{i\varphi}\lambda_1(1 \pm \sqrt{\Delta}) \\ \mathcal{N}_\pm^2 &= (1 - \lambda_0^2 \sin^2 r - \lambda_1^2) z_\pm^2 + |y_\pm|^2. \end{aligned} \tag{9}$$

It is easy to show $\langle a_+|a_-\rangle = 0$, which guarantees that ρ_{IBC} is a quantum state.

Since the three-tangle for mixed states is defined via a convex roof method[8], we should find an optimal decomposition of ρ_{IBC} for analytical computation of the three-tangle. It is to be noted that the three-tangles for $|a_\pm\rangle$ are

$$\tau_3\left(|a_\pm\rangle\right) = 4\lambda_0^2\lambda_4^2\cos^2 r\left(\frac{z_\pm}{\mathcal{N}_\pm}\right)^4. \tag{10}$$

Now, we define

$$|F,\theta\rangle = \sqrt{p}|a_+\rangle + e^{i\theta}\sqrt{1-p}|a_-\rangle. \tag{11}$$

Then, it is easy to show that ρ_{IBC} can be represented as

$$\rho_{IBC} = \frac{1}{2}|F,\theta\rangle\langle F,\theta| + \frac{1}{2}|F,\theta+\pi\rangle\langle F,\theta+\pi| \tag{12}$$

and the three-tangle of $|F,\theta\rangle$ is

$$\tau_3\left(|F,\theta\rangle\right) = 4\lambda_0^2\lambda_4^2\cos^2 r\left[\frac{pz_+^2}{\mathcal{N}_+^2} + \frac{(1-p)z_-^2}{\mathcal{N}_-^2} + \frac{2\sqrt{p(1-p)}z_+z_-}{\mathcal{N}_+\mathcal{N}_-}\cos\theta\right]^2. \tag{13}$$

Now, we assume that Eq. (12) is an optimal decomposition for the three-tangle for the time being. Then, the three-tangle of ρ_{IBC} is

$$\tau_3\left(\rho_{IBC}\right) = 4\lambda_0^2\lambda_4^2\cos^2 r\left[\left(\frac{pz_+^2}{\mathcal{N}_+^2} + \frac{(1-p)z_-^2}{\mathcal{N}_-^2}\right)^2 + \frac{4p(1-p)z_+^2z_-^2}{\mathcal{N}_+^2\mathcal{N}_-^2}\cos^2\theta\right]. \tag{14}$$

Therefore, the convex-roof constraint of the three-tangle leads to θ being fixed as $\theta = \pi/2$. Consequently, Eq. (14) indicates that $\tau_3\left(\rho_{IBC}\right)$ is really a convex function with respect to p. Therefore, Eq. (12) is really an optimal decomposition of ρ_{IBC} if $\theta = \pi/2$. By inserting Eq. (7) and Eq. (9) into Eq. (14) and imposing $\theta = \pi/2$, it is straight to show that

$$\tau_3\left(\rho_{IBC}\right) = 4\lambda_0^2\lambda_4^2\cos^2 r. \tag{15}$$

Thus, Eq. (4) holds when Alice is chosen as the accelerating party.

Now, we choose Bob as the accelerating party. Following a procedure similar to the one described above, one can show that ρ_{AIC} becomes

$$\rho_{AIC} = p|b_+\rangle\langle b_+| + (1-p)|b_-\rangle\langle b_-| \tag{16}$$

where $p = (1+\sqrt{\sigma})/2$ with

$$\sigma = \left[1 - 2(\lambda_0^2+\lambda_1^2+\lambda_2^2)\sin^2 r\right]^2 + 4\sin^2 r\left(\lambda_1^2\lambda_3^2 + \lambda_2^2\lambda_4^2 + 2\lambda_1\lambda_2\lambda_3\lambda_4\cos\varphi\right). \tag{17}$$

In Eq. (16) the vectors $|b_\pm\rangle$ are given by

$$|b_\pm\rangle = \frac{1}{\mathcal{N}_\pm}\left[a^\pm_{000}|000\rangle + a_{010}|010\rangle + a^\pm_{100}|100\rangle + a^\pm_{101}|101\rangle + a^\pm_{110}|110\rangle + a^\pm_{111}|111\rangle\right] \tag{18}$$

where

$$\begin{aligned} a^\pm_{000} &= \lambda_0\cos r\left[1-2\sin^2 r(\lambda_0^2+\lambda_1^2+\lambda_2^2)\pm\sqrt{\sigma}\right] \\ a_{010} &= 2\lambda_0\sin^2 r\left[e^{-i\varphi}\lambda_1\lambda_3+\lambda_2\lambda_4\right] \\ a^\pm_{100} &= e^{i\varphi}\lambda_1\cos r\left[1-2\sin^2 r(\lambda_0^2+\lambda_1^2+\lambda_2^2)\pm\sqrt{\sigma}\right] \\ a^\pm_{101} &= \lambda_2\cos r\left[1-2\sin^2 r(\lambda_0^2+\lambda_1^2+\lambda_2^2)\pm\sqrt{\sigma}\right] \\ a^\pm_{110} &= \lambda_3\left[1-2\sin^2 r(\lambda_0^2+\lambda_2^2)\pm\sqrt{\sigma}\right]+2e^{i\varphi}\lambda_1\lambda_2\lambda_4\sin^2 r \\ a^\pm_{111} &= \lambda_4\left[1-2\sin^2 r(\lambda_0^2+\lambda_1^2)\pm\sqrt{\sigma}\right]+2e^{-i\varphi}\lambda_1\lambda_2\lambda_3\sin^2 r \end{aligned} \tag{19}$$

and $\mathcal{N}_\pm^2 = |a^\pm_{000}|^2 + |a_{010}|^2 + |a^\pm_{100}|^2 + |a^\pm_{101}|^2 + |a^\pm_{110}|^2 + |a^\pm_{111}|^2$. It is easy to show that the three-tangles of $|b_\pm\rangle$ are

$$\tau_3\left(|b_\pm\rangle\right) = 4\lambda_0^2\lambda_4^2\cos^2 r\left[\frac{1-2\sin^2 r(\lambda_0^2+\lambda_1^2+\lambda_2^2)\pm\sqrt{\sigma}}{\mathcal{N}_\pm}\right]^4. \tag{20}$$

In order to compute the three-tangle of ρ_{AIC} we define

$$|G,\theta\rangle = \sqrt{p}|b_+\rangle + e^{i\theta}\sqrt{1-p}|b_-\rangle. \tag{21}$$

Then, ρ_{AIC} can be represented as

$$\rho_{AIC} = \frac{1}{2}|G,\theta\rangle\langle G,\theta| + \frac{1}{2}|G,\theta+\pi\rangle\langle G,\theta+\pi| \tag{22}$$

and the three-tangle of $|G,\theta\rangle$ is

$$\tau_3\left(|G,\theta\rangle\right) = 4\lambda_0^2\lambda_4^2\cos^2 r\left[(X-Y)^2+Z^2+4XY\cos^2\theta-2Z(X+Y)\cos\theta\right] \tag{23}$$

where

$$\begin{aligned} X &= \frac{p}{\mathcal{N}_+^2}\left[1-2\sin^2 r(\lambda_0^2+\lambda_1^2+\lambda_2^2)+\sqrt{\sigma}\right]^2 \\ Y &= \frac{1-p}{\mathcal{N}_-^2}\left[1-2\sin^2 r(\lambda_0^2+\lambda_1^2+\lambda_2^2)-\sqrt{\sigma}\right]^2 \\ Z &= 8\sin^2 r\frac{\sqrt{p(1-p)}}{\mathcal{N}_+\mathcal{N}_-}\left[\lambda_1^2\lambda_3^2+\lambda_2^2\lambda_4^2+2\lambda_1\lambda_2\lambda_3\lambda_4\cos\varphi\right]. \end{aligned} \tag{24}$$

Therefore, if Eq. (22) is the optimal decomposition, the three-tangle of ρ_{AIC} is

$$\tau_3\left(\rho_{AIC}\right) = 4\lambda_0^2\lambda_4^2\cos^2 r\left[(X-Y)^2 + Z^2 + 4XY\cos^2\theta\right]. \tag{25}$$

Since the three-tangle is defined as a convex roof method, we should choose $\theta = \pi/2$, which gives

$$\tau_3\left(\rho_{AIC}\right) = 4\lambda_0^2\lambda_4^2\cos^2 r\left[(X-Y)^2 + Z^2\right]. \tag{26}$$

It is easy to show that Eq. (26) is really a convex function with respect to p. Using of Eq. (17), Eq. (19), and Eq. (24) and performing a series of calculations, we can show that $(X-Y)^2 + Z^2 = 1$, which results in

$$\tau_3\left(\rho_{AIC}\right) = 4\lambda_0^2\lambda_4^2\cos^2 r. \tag{27}$$

Thus, Eq. (4) holds when Bob is chosen as the accelerating party.

Finally, let us choose Charlie as the accelerating party. Since $|\psi\rangle_{ABC}$ given in Eq. (3) has Bob $\leftrightarrow$ Charlie and $\lambda_2 \leftrightarrow \lambda_3$ symmetry, the previous calculation implies

$$\tau_3(\rho_{ABI}) = 4\lambda_0^2\lambda_4^2\cos^2 r. \tag{28}$$

Thus, Eq. (4) holds regardless of the choice of the accelerating party.

So far, we have shown that the canonical form of the three-qubit state (3) obeys Eq. (4) in the non-inertial frame. However, this does not mean that the arbitrary three-qubit pure state $|\psi_3\rangle = \sum_{i,j,k=0}^{1} a_{ijk}|ijk\rangle$ obeys Eq. (4). The question arises as to whether the Unruh transformation (1) that is taken before the appropriate Schmidt decomposition may generate a result different from from Eq. (4). However, this is not the case; the following two theorems show that $|\psi_3\rangle$ also obeys Eq. (4) in the non-inertial frame.

Theorem 1. *Let Alice and Bob initially share the arbitrary fermionic two-qubit pure state $|\psi_2\rangle_{AB} = \sum_{i,j=0}^{1} a_{ij}|ij\rangle$, whose concurrence is $\tau_2^{(0)}$. If one party accelerates with respect to the other party, then the concurrence reduces to $\tau_2^{(0)}\cos r$.*

Proof. It is to be noted that the concurrence of $|\psi_2\rangle_{AB}$ is

$$\tau_2^{(0)} = 2|a_{00}a_{11} - a_{01}a_{10}|. \tag{29}$$

First, we choose Bob as the accelerating party. Then, the Unruh transformation (1) and the partial trace over II gives

$$\rho_{AI} = \begin{pmatrix} |a_{00}|^2 \cos^2 r & a_{00}a_{01}^* \cos r & a_{00}a_{10}^* \cos^2 r & a_{00}a_{11}^* \cos r \\ a_{00}^* a_{01} \cos r & |a_{01}|^2 + |a_{00}|^2 \sin^2 r & a_{01}a_{10}^* \cos r & a_{01}a_{11}^* + a_{00}a_{10}^* \sin^2 r \\ a_{00}^* a_{10} \cos^2 r & a_{01}^* a_{10} \cos r & |a_{10}|^2 \cos^2 r & a_{10}a_{11}^* \cos r \\ a_{00}^* a_{11} \cos r & a_{01}^* a_{11} + a_{00}^* a_{10} \sin^2 r & a_{10}^* a_{11} \cos r & |a_{11}|^2 + |a_{10}|^2 \sin^2 r \end{pmatrix}. \tag{30}$$

Using Eq. (30) we can construct $R = \rho_{AI}(\sigma_y \otimes \sigma_y)\rho_{AI}^*(\sigma_y \otimes \sigma_y)$. Although R is a complicated matrix, it is possible to compute the eigenvalues analytically by solving $\det(R - \lambda I) = 0$. The eigenvalues of R are $\{0, 0, 0, 4|a_{00}a_{11} - a_{01}a_{10}|^2 \cos^2 r\}$. Therefore, by making use of the Wootters formula[9], we obtain the concurrence of ρ_{AI} as

$$\tau_2(\rho_{AI}) = 2|a_{00}a_{11} - a_{01}a_{10}| \cos r = \tau_2^{(0)} \cos r. \tag{31}$$

Since $\tau_2^{(0)}$ has $a_{01} \leftrightarrow a_{10}$ symmetry, the choice of Alice as the accelerating party leads to the same result, which completes the proof.

Now, we state the main theorem of the paper.

Theorem 2. *Let Alice, Bob, and Charlie initially share the arbitrary fermionic three-qubit pure state* $|\psi_3\rangle_{ABC} = \sum_{i,j,k=0}^{1} a_{ijk}|ijk\rangle$*, whose three-tangle is* $\tau_3^{(0)}$*. If one party accelerates with respect to other parties, then the three-tangle reduces to* $\tau_3^{(0)} \cos^2 r$*.*

Proof. It is to be noted that the three-tangle of $|\psi_3\rangle$[6] is

$$\tau_3^{(0)} = 4|d_1 - 2d_2 + 4d_3|, \tag{32}$$

where

$$\begin{aligned} d_1 &= a_{000}^2 a_{111}^2 + a_{001}^2 a_{110}^2 + a_{010}^2 a_{101}^2 + a_{100}^2 a_{011}^2, \\ d_2 &= a_{000}a_{111}a_{011}a_{100} + a_{000}a_{111}a_{101}a_{010} + a_{000}a_{111}a_{110}a_{001} \\ &\quad + a_{011}a_{100}a_{101}a_{010} + a_{011}a_{100}a_{110}a_{001} + a_{101}a_{010}a_{110}a_{001}, \\ d_3 &= a_{000}a_{110}a_{101}a_{011} + a_{111}a_{001}a_{010}a_{100}. \end{aligned} \tag{33}$$

Since $\tau_3^{(0)}$ is permutation-invariant, it is sufficient to provide proof for the case that Alice is chosen as the accelerating party. We provide the proof via two different methods. The first proof is simple but intuitive while the second is lengthy and direct. Thus, we present the second one schematically.

As shown in Ref. [6] the three-tangle is defined via the monogamy inequality $\mathcal{C}_{AB}^2+\mathcal{C}_{AC}^2 \leq \mathcal{C}_{A(BC)}^2$, where $\mathcal{C} = \tau_2$ is a concurrence. Therefore, the three-tangle of the mixed state can be written as

$$\tau_3 = \min\left[\mathcal{C}_{A(BC)}^2 - \mathcal{C}_{AB}^2 - \mathcal{C}_{AC}^2\right], \tag{34}$$

where the minimum is taken over all possible decompositions of the given mixed state. Since Alice is chosen as the accelerating party, theorem 1 implies that each concurrence in Eq. (34) has a degradation factor $\cos r$. Therefore, Eq. (34) implies that the three-tangle has a degradation factor $\cos^2 r$, which is what theorem 2 states.

Another method to prove theorem 2 is similar to the method for the proof of theorem 1. After performing the Unruh transformation (1) on Alice's qubit of $|\psi_3\rangle_{ABC}$ and taking a partial trace over II, one can derive ρ_{IBC} straightforwardly. Although ρ_{IBC} is an extremely complicated 8×8 matrix, one can show from a purification protocol that its rank is only 2. Therefore, it is possible to derive the spectral decomposition of ρ_{IBC} as a form $\rho_{IBC} = p|\mu_+\rangle\langle\mu_+|+(1-p)|\mu_-\rangle\langle\mu_-|$. Consequently, following a procedure similar to the one we used previously, we can compute the three-tangle of ρ_{IBC} explicitly. We perform this calculation by making use of the software Mathematica, and we finally arrive at Eq. (4) again, which completes the proof.

In this paper we investigated the degradation of the tripartite fermionic entanglement in a non-inertial frame. If the given three parties initially share a pure state, the degradation factor is shown to be simply $\cos^2 r$ regardless of the initial state and choice of the accelerating party. This is a surprising result in the sense of the simpleness of Eq. (4).

It is natural to ask whether or not the simpleness of Eq. (4) is maintained when the initial state is a mixed state. Another natural question is to ask whether or not the simpleness of Eq. (4) is maintained beyond the single mode approximation. We believe that Eq. (4) or other such simple expressions are valid in these cases. However, our speculation requires to be confirmed via appropriate calculations.

Another question that arises is the extension of Eq. (4) to multipartite entanglement. If an n-tangle is constructed in the future, we speculate that the degradation factor in a non-inertial frame would be $\cos^{\alpha_n} r$, where $\alpha_n = 2^{n-2}$. However, there are several obstacles to confirming this hypothesis. Above all, the explicit expression of n-tangle is not yet known. Further, there is no calculational technique for the computation of the n-tangle of n-qubit mixed states. Our future studies will further explore these issues

Acknowledgement: This research was supported by the Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology(2011-0011971).

---

[1] A. Peres and D. R. Terno, Rev. Mod. Phys. **76** (2004) 93 [quant-ph/0212023]; A. Peres and D. R. Terno, Int. J. Quant. Info. bf 1 (2003) 225 [quant-ph/0301065]; A. Peres, Fortsch. Phys. **52** (2004) 1052 [quant-ph/0405127].

[2] M. Czachor, Phys. Rev. **A 55** (1997) 72 [quant-ph/9609022]; A. Peres, P. F. Scudo and R. Terno, Phys. Rev. Lett. **88** (2002) 230402 [quant-ph/0203033]; P. M. Alsing and G. J. Milburn, Quantum Inf. Comput. **2** (2002) 487 [quant-ph/0203051]; R. M. Gingrich and C. Adami, Phys. Rev. Lett. **89** (2002) 270402 [quant-ph/0205179].

[3] I. Fuentes-Schuller and R. B. Mann, Phys. Rev. Lett. **95** (2005) 120404 [quant-ph/0410172]; P. M. Alsing, I. Fuentes-Schuller, R. B. Mann, and T. E. Tessier, Phys. Rev. **A 74** (2006) 032326 [quant-ph/0603029]; Y. Ling, S. He, W. Qiu and H. Zhang, J. Phys, **A40** (2007) 9025 [quant-ph/0608029]; Q. Pan and J. Jing, Phys. Rev. **A 77** (2008) 024302 [arXiv:08021238 (quant-ph)]; M. R. Hwang, D. K. Park, and E. Jung, Phys. Rev. **A 83** (2011) 012111 [arXiv:1010.6154 (hep-th)]; E. Martin-Martinez and I. Fuentes, Phys. Rev. **A 83** (2011) 052306 [arXiv:1102.4759 (quant-ph)]; M. Montero and E. Martin-Martinez, Phys. Rev. **A 83** (2011) 062323 [arXiv:1104.2307 (quant-ph)]; M. Montero, J. Leon, and E. Martin-Martinez, Phys. Rev. **A 84** (2011) 042320 [arXiv:1108.1111 (quant-ph)].

[4] S. B. Giddings, arXiv:1201.1037 (quant-ph) and references therein.

[5] W. G. Unruh, Phys. Rev. **D 14** (1976) 670; N. D. Birrel and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).

[6] V. Coffman, J. Kundu and W. K. Wootters, Phys. Rev. **A 61** (2000) 052306 [quant-ph/9907047].

[7] A. Acin *et al*, Phys. Rev. Lett. **85** (2000) 1560 [quant-ph/0003050].

[8] C. H. Bennett, D. P. DiVincenzo, J. A. Smokin and W. K. Wootters, Phys. Rev. **A 54** (1996) 3824; A. Uhlmann, Phys. Rev. **A 62** (2000) 032307.

[9] S. Hill and W. K. Wootters, Phys. Rev. Lett. **78** (1997) 5022 [quant-ph/9703041]; W. K. Wootters, Phys. Rev. Lett. **80** (1998) 2245 [quant-ph/9709029].